AKCE Int. J. Graphs Comb., 10, No. 2 (2013), pp. 147-155

On the total edge irregularity strength of generalized helm

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Abstract

A total k-labeling is a map that carries vertices and edges of a graph G into a set of positive integer labels $\{1, 2, ..., k\}$. An edge irregular total k-labeling of a graph G is a total k-labeling such that the weights calculated for all edges are distinct. The weight of an edge uv in G is defined as the sum of the label of u, the label of v and the label of uv. The total edge irregularity strength of G, denoted by tes(G), is the minimum value of the largest label k over all such edge irregular total k-labelings. In this paper, we investigate the total edge irregularity strength of generalized helm, H_n^m for $n \geq 3, m = 1, 2,$ and $m \equiv 0$ (mod 3).

Keywords: total k-labeling, edge irregular total k -labeling, total edge irregularity strength, generalized helm.

2010 Mathematics Subject Classification: 05C78.

1. Introduction

Let $G(V, E)$ be a connected, simple and undirected graph with vertex set V and edge set E. Wallis [10] defined a labeling (or valuation) of graph as follows: A labeling of a graph is a map that carries graph elements to the numbers (usually to the positive or non negative integers). The most common choices of domain are the set of all vertices and edges (such a labeling is called a total labeling), the vertex set alone (called a vertex labeling) or the edge set alone (called an edge labeling). In the recent development, the graph labeling is also defined as various functions (see Gallian [4]).

For a graph $G(V,E)$, Bača, *et al.* [3] defined a labeling $f: V \cup E \to \{1,2,...,k\}$ to be a total k-labeling. An edge irregular total k-labeling of a graph G is a total k-labeling such that the weights calculated at all edges are distinct. The weight of an edge uv in G, denoted by $wt(uv)$, is defined as the sum of the label of u, the label of v and the label of uv, that is

$$wt(uv) = f(u) + f(uv) + f(v).$$

They also defined the total edge irregularity strength of G, denoted by $\text{tes}(G)$, as the minimum value of the largest label k over all such edge irregular total k-labelings.

The total edge irregularity strength for various classes of graphs have been determined. For instances, Bača, *et al.* [3] gave a lower bound and an upper bound on total edge irregularity strength for any graph G with vertex set V and a non-empty edge set E, $\left\lceil \frac{|E|+2}{3} \right\rceil \leq \text{tes}(G) \leq |E|$. They also gave a lower bound of total edge irregularity strength for a graph G with maximum degree Δ, $\left\lceil \frac{\Delta+1}{2} \right\rceil \leq \text{tes}(G)$. In the same paper, they proved the total edge irregularity strength of paths, cycles, stars, wheels and friendship graphs, that are, $\text{tes}(P_n) = \text{tes}(C_n) = \left\lceil \frac{n+2}{3} \right\rceil$, $\text{tes}(K_{1,n}) = \left\lceil \frac{n+1}{2} \right\rceil$, $\text{tes}(W_n) = \left\lceil \frac{2n+2}{3} \right\rceil$ for $n \geq 3$, $\text{tes}(F_n) = \left\lceil \frac{3n+2}{3} \right\rceil$ respectively.

Furthermore, the total edge irregularity strength of tree G with edge set E and maximum degree Δ had been found by Ivančo and Jendrol [7], that is $\text{tes}(G) = \max\left\{ \left\lceil \frac{\Delta+1}{2} \right\rceil, \left\lceil \frac{|E|+2}{3} \right\rceil \right\}$. According to this result, then the total edge irregularity strength of subdivision of stars, S_n^m, is $\left\lceil \frac{(m+1)n+2}{3} \right\rceil$, as Siddiqui [9] had done for $n \geq 3$ and $1 \leq m \leq 8$. Nurdin *et al.* [8] proved the total edge irregularity strength of the corona product of paths with some graphs, namely paths, cycles, stars, gears, friendships and wheels. The total edge irregularity strength of the categorical product and strong product of two paths can be found in [1] and [2]. Haque [5] investigated the total edge irregularity strength of generalized Petersen graphs $P(n,k)$ and proved that $\text{tes}(P(n,k)) = n+1$ for $k \neq n/2$ and $\text{tes}(P(n,k)) = \left\lceil \frac{5n+4}{6} \right\rceil$ for $k = n/2$. Indriati, *et al.* [6] determined the total edge irregularity strength of helm, H_n, and disjoint union of t isomorphic helms, tH_n, and found that $\text{tes}(H_n) = n+1$, while $\text{tes}(tH_n) = tn+1$. In this paper, we investigate the total edge irregularity strength of generalized helm, H_n^m for $n \geq 3$, $m = 1$, 2 and $m \equiv 0 \ (mod\ 3)$.

2. Main Results

A generalized helm, H_n^m, is a graph obtained by inserting m vertices to every pendant edge of helm H_n. A generalized helm H_n^m has $(m+2)n+1$ vertices and $(m+3)n$ edges. Let the vertex set of H_n^m be $V(H_n^m) = \{v_{i,j} : 1 \leq i \leq n, 1 \leq j \leq m+1\} \cup \{u_i : 1 \leq i \leq n\} \cup \{w\}$ and the edge set of H_n^m be $E(H_n^m) = \{(v_{i,j}v_{i,j+1}) : 1 \leq i \leq n, 1 \leq j \leq m\} \cup \{(v_{i,m+1}u_i) :$

$1 \le i \le n\} \cup \{(u_i u_{i+1(\text{mod } n)}) : 1 \le i \le n\} \cup \{(wu_i) : 1 \le i \le n\}$. Figure 1 illustrates the generalized helm H_n^m.

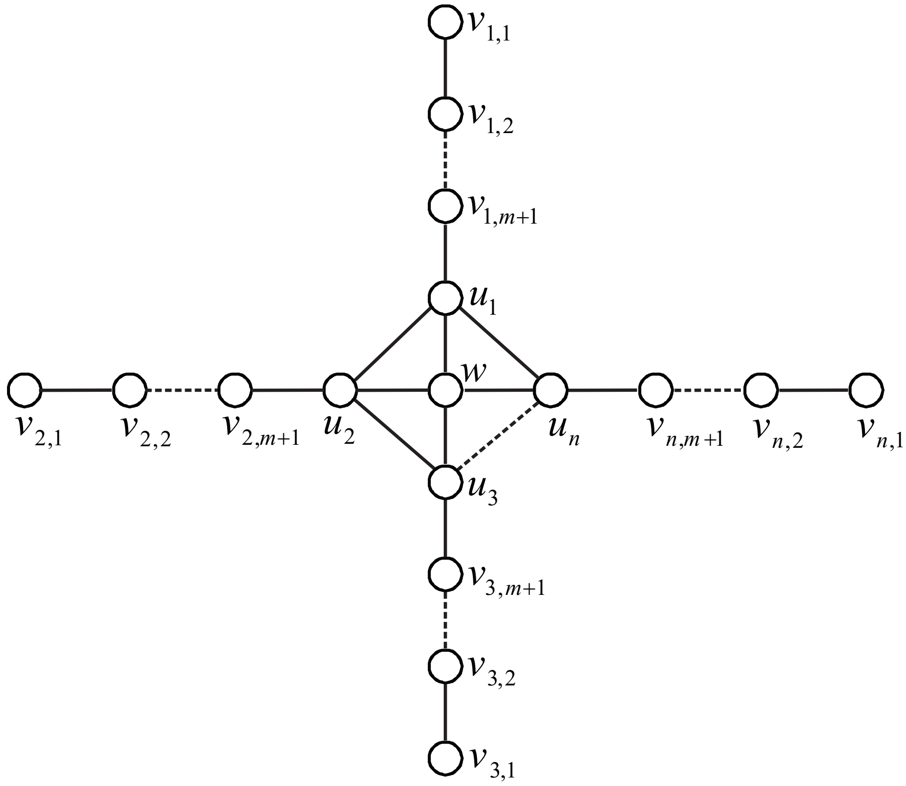

Figure 1: The Generalized Helm H_n^m

In the next theorem, we present the total edge irregularity strength of generalized helm H_n^1 for $n \ge 3$ as follows:

Theorem 2.1. *For $n \ge 3$, $tes(H_n^1) = \left\lceil \frac{4n+2}{3} \right\rceil$.*

Proof. From the lower bound of total edge irregularity strength we have that $\text{tes}(H_n^1) \ge \left\lceil \frac{4n+2}{3} \right\rceil$, $n \ge 3$. To prove the equality, it is sufficient to show the existence of an edge irregular total k_1-labeling with $k_1 = \left\lceil \frac{4n+2}{3} \right\rceil$.

Let $V(H_n^1) = \{u_i, v_{i,1}, v_{i,2} : 1 \le i \le n\} \cup \{w\}$ be the vertex set and $E(H_n^1) = \{wu_i, u_i v_{i,2}, v_{i,2} v_{i,1} : 1 \le i \le n\} \cup \{u_i u_{i+1} : 1 \le i \le n-1\} \cup \{u_n u_1\}$ be the edge set of H_n^1. Define a total labeling f_1 in the following way:

Case 1. For $n \equiv 0$ $(mod\ 3)$ and $n \equiv 2$ $(mod\ 3)$; $n \ge 3$.

$$f_1(v_{i,1}) = 1, i = 1, \dots, n,$$
$$f_1(v_{i,2}) = \begin{cases} i, & \text{for } 1 \le i \le \left\lceil \frac{2n+3}{3} \right\rceil, \\ \left\lceil \frac{2n+3}{3} \right\rceil, & \text{for} \left\lceil \frac{2n+3}{3} \right\rceil + 1 \le i \le n, \end{cases}$$
$$f_1(u_i) = \begin{cases} n+1, & \text{for } 1 \le i \le \left\lceil \frac{2n+3}{3} \right\rceil, \\ i - \left\lceil \frac{2n}{3} \right\rceil + n, & \text{for } \left\lceil \frac{2n+3}{3} \right\rceil + 1 \le i \le n, \end{cases}$$
$$f_1(w) = \left\lceil \frac{4n+2}{3} \right\rceil,$$
$$f_1(v_{i,1}v_{i,2}) = \begin{cases} 1, & \text{for } 1 \le i \le \left\lceil \frac{2n+3}{3} \right\rceil, \\ i - \left\lceil \frac{2n}{3} \right\rceil, & \text{for } \left\lceil \frac{2n+3}{3} \right\rceil + 1 \le i \le n, \end{cases}$$
$$f_1(u_i v_{i,2}) = 1, i = 1, \dots, n,$$
$$f_1(u_i u_{i+1(mod\ n)}) = \begin{cases} i, & \text{for } 1 \le i \le \left\lceil \frac{2n}{3} \right\rceil, \\ 2\left\lceil \frac{2n}{3} \right\rceil + 1 - i, & \text{for } \left\lceil \frac{2n}{3} \right\rceil + 1 \le i \le n-1, \\ \left\lceil \frac{2n}{3} \right\rceil + 1, & \text{for } i = n, \end{cases}$$
$$f_1(wu_i) = \begin{cases} \left\lceil \frac{2n-1}{3} \right\rceil + i, & \text{for } 1 \le i \le \left\lceil \frac{2n+3}{3} \right\rceil, \\ \left\lceil \frac{4n+2}{3} \right\rceil, & \text{for } \left\lceil \frac{2n+3}{3} \right\rceil + 1 \le i \le n. \end{cases}$$

Case 2. For $n \equiv 1$ $(mod\ 3)$; $n \ge 3$.

$$f_1(v_{i,1}) = 1, i = 1, \dots, n,$$
$$f_1(v_{i,2}) = \begin{cases} i, & \text{for } 1 \le i \le \left\lceil \frac{2n}{3} \right\rceil, \\ \left\lceil \frac{2n}{3} \right\rceil, & \text{for } \left\lceil \frac{2n}{3} \right\rceil + 1 \le i \le n, \end{cases}$$
$$f_1(u_i) = \begin{cases} n+1, & \text{for } 1 \le i \le \left\lceil \frac{2n}{3} \right\rceil, \\ i - \left\lceil \frac{2n}{3} \right\rceil + n + 1, & \text{for } \left\lceil \frac{2n}{3} \right\rceil + 1 \le i \le n, \end{cases}$$
$$f_1(w) = \left\lceil \frac{4n+2}{3} \right\rceil,$$
$$f_1(v_{i,1}v_{i,2}) = \begin{cases} 1, & \text{for } 1 \le i \le \left\lceil \frac{2n}{3} \right\rceil, \\ i - \left\lceil \frac{2n}{3} \right\rceil + 1, & \text{for } \left\lceil \frac{2n}{3} \right\rceil + 1 \le i \le n, \end{cases}$$
$$f_1(u_i v_{i,2}) = 1, i = 1, \dots, n,$$

$$f_1(u_iu_{i+1(mod\ n)}) = \begin{cases} i, & \text{for } 1 \le i \le \left\lceil \frac{2n}{3} \right\rceil - 1, \\ 2\left\lceil \frac{2n}{3} \right\rceil - 1 - i, & \text{for } \left\lceil \frac{2n}{3} \right\rceil \le i \le n-1, \\ \left\lceil \frac{2n}{3} \right\rceil, & \text{for } i = n, \end{cases}$$

$$f_1(wu_i) = \begin{cases} \left\lceil \frac{2n}{3} \right\rceil + i, & \text{for } 1 \le i \le \left\lceil \frac{2n}{3} \right\rceil - 1, \\ \left\lceil \frac{4n+2}{3} \right\rceil, & \text{for } \left\lceil \frac{2n}{3} \right\rceil \le i \le n. \end{cases}$$

It can be seen that the function f_1 is a map from $V(H_n^1) \cup E(H_n^1)$ into $\left\{1, 2, \ldots, \left\lceil \frac{4n+2}{3} \right\rceil\right\}$.

Thus, f_1 is a total k_1-labeling with $k_1 = \left\lceil \frac{4n+2}{3} \right\rceil$.

By observation, the weights of the edges are:

$$\begin{aligned} wt(v_{i,1}v_{i,2}) &= 2 + i, i = 1, \ldots, n, \\ wt(u_iv_{i,2}) &= n + 2 + i, i = 1, \ldots, n, \\ wt(u_iu_{i+1(\text{mod n})}) &= 2n + 2 + i, i = 1, \ldots, n, \\ wt(wu_i) &= 3n + 2 + i, i = 1, \ldots, n. \end{aligned}$$

It can be seen that the weights of edges of H_n^1 under the total k_1-labeling, f_1, form consecutive integers from 3 up to $4n + 2$. It means that the weights of all edges are distinct. So, the labeling is an edge irregular total k_1-labeling with $k_1 = \lceil \frac{4n+2}{3} \rceil$. Therefore $\text{tes}(H_n^1) = \lceil \frac{4n+2}{3} \rceil$. □

Next, we continue to find the total edge irregularity strength of H_n^2.

Theorem 2.2. *For* $n \ge 3$, $tes(H_n^2) = \left\lceil \frac{5n+2}{3} \right\rceil$.

Proof. From [3] we know that $\text{tes}(H_n^2) \ge \left\lceil \frac{5n+2}{3} \right\rceil$, $n \ge 3$. To prove the equality, next we show the existence of an edge irregular total k_2-labeling with $k_2 = \left\lceil \frac{5n+2}{3} \right\rceil$.

Let $V(H_n^2) = \{v_{i,j} : 1 \le i \le n, 1 \le j \le 3\} \cup \{u_i : 1 \le i \le n\} \cup \{w\}$ be the vertex set and $E(H_n^2) = \{(v_{i,j}v_{i,j+1}) : 1 \le i \le n, 1 \le j \le 2\} \cup \{(v_{i,3}u_i) : 1 \le i \le n\} \cup \{(u_iu_{i+1(\text{mod } n)}) : 1 \le i \le n\} \cup \{(wu_i) : 1 \le i \le n\}$ be the edge set of H_n^2. Define a total labeling f_2 in the following way:

$$\begin{aligned} f_2(v_{i,1}) &= 1,\ i = 1, \ldots, n, \\ f_2(v_{i,2}) &= i,\ i = 1, \ldots, n, \\ f_2(v_{i,3}) &= n + 1,\ i = 1, \ldots, n, \end{aligned}$$

$$f_2(u_i) = \begin{cases} n+i, & \text{for } 1 \le i \le \left\lceil \frac{2n+2}{3} \right\rceil, \\ \left\lceil \frac{5n+2}{3} \right\rceil, & \text{for } \left\lceil \frac{2n+2}{3} \right\rceil + 1 \le i \le n, \end{cases}$$

$$f_2(w) = \left\lceil \frac{5n+2}{3} \right\rceil,$$

$$f_2(v_{i,1}v_{i,2}) = f_2(v_{i,2}v_{i,3}) = 1, \; i = 1, \dots, n,$$

$$f_2(v_{i,3}u_i) = \begin{cases} 1, & \text{for } 1 \le i \le \left\lceil \frac{2n+2}{3} \right\rceil, \\ i - \left\lceil \frac{2n+2}{3} \right\rceil + 1, & \text{for } \left\lceil \frac{2n+2}{3} \right\rceil + 1 \le i \le n, \end{cases}$$

$$f_2(u_iu_{i+1(\text{mod } n)}) = \begin{cases} n+1-i, & \text{for } 1 \le i \le \left\lceil \frac{2n-1}{3} \right\rceil, \\ i - \left\lceil \frac{2n-1}{3} \right\rceil + \left\lceil \frac{n-1}{3} \right\rceil, & \text{for } \left\lceil \frac{2n-1}{3} \right\rceil + 1 \le i \le n-1, \\ \left\lceil \frac{4n-1}{3} \right\rceil, & \text{for } i = n, \end{cases}$$

$$f_2(wu_i) = \begin{cases} \left\lceil \frac{4n+2}{3} \right\rceil, & \text{for } 1 \le i \le \left\lceil \frac{2n+2}{3} \right\rceil, \\ \left\lceil \frac{4n+2}{3} \right\rceil - \left\lceil \frac{2n+2}{3} \right\rceil + i, & \text{for } \left\lceil \frac{2n+2}{3} \right\rceil + 1 \le i \le n. \end{cases}$$

It can be seen that the function f_2 is a map from $V(H_n^2) \cup E(H_n^2)$ into $\left\{1, 2, \dots, \left\lceil \frac{5n+2}{3} \right\rceil \right\}$. Thus, f_2 is a total k_2-labeling with $k_2 = \left\lceil \frac{5n+2}{3} \right\rceil$.
Observe that

$$\begin{aligned} wt(v_{i,1}v_{i,2}) &= 2+i, \; i = 1, \dots, n, \\ wt(v_{i,2}v_{i,3}) &= n+2+i, \; i = 1, \dots, n, \\ wt(v_{i,3}u_i) &= 2n+2+i, \; i = 1, \dots, n, \\ wt(u_iu_{i+1(\text{mod } n)}) &= 3n+2+i, \; i = 1, \dots, n, \\ wt(wu_i) &= 4n+2+i, \; i = 1, \dots, n. \end{aligned}$$

So the weights of edges of H_n^2 under the total k_2-labeling, f_2, form consecutive integers from 3 up to $5n+2$. It means that the weights of all edges are distinct. So, the labeling is an edge irregular total k_2-labeling with $k_2 = \left\lceil \frac{5n+2}{3} \right\rceil$. Therefore tes$(H_n^2) = \left\lceil \frac{5n+2}{3} \right\rceil$. □

We give an example of an edge irregular total labeling of H_4^2 in Figure 2.

The following theorem shows the total edge irregularity strength of H_n^m for $m \equiv 0 \pmod 3$. Let m_0 be the notation of $m \equiv 0 \pmod 3$.

Theorem 2.3. *For $n \ge 3$, $tes(H_n^{m_0}) = \left\lceil \frac{(m_0+3)n+2}{3} \right\rceil$.*

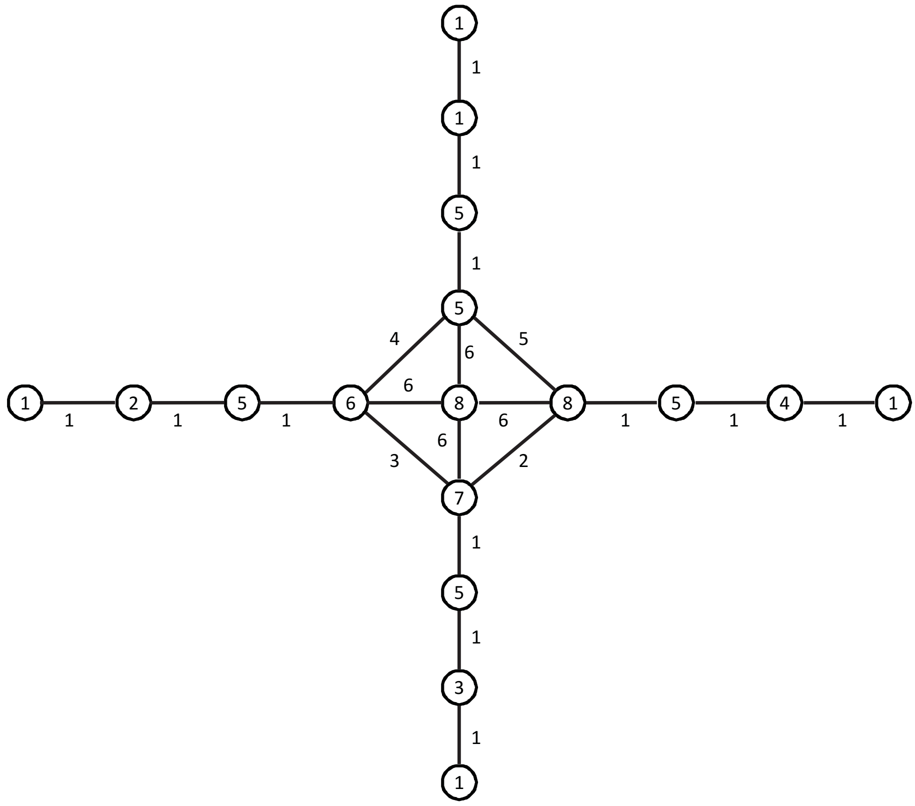

Figure 2: An edge irregular total 8-labeling of H_4^2

Proof. From the lower bound of total edge irregularity strength we have that $\text{tes}(H_n^{m_0}) \geq \left\lceil \frac{(m_0+3)n+2}{3} \right\rceil$, $n \geq 3$. To prove that $\text{tes}(H_n^{m_0}) = \left\lceil \frac{(m_0+3)n+2}{3} \right\rceil$, next we show the existence of an edge irregular total k_{m_0}-labeling with $k_{m_0} = \left\lceil \frac{(m_0+3)n+2}{3} \right\rceil$.

Let $V(H_n^{m_0}) = \{v_{i,j} : 1 \leq i \leq n, 1 \leq j \leq m_0 + 1\} \cup \{u_i : 1 \leq i \leq n\} \cup \{w\}$ be the vertex set and $E(H_n^{m_0}) = \{(v_{i,j}v_{i,j+1}) : 1 \leq i \leq n, 1 \leq j \leq m_0\} \cup \{(v_{i,m_0+1}u_i) : 1 \leq i \leq n\} \cup \{(u_iu_{i+1(\text{mod } n)}) : 1 \leq i \leq n\} \cup \{(wu_i) : 1 \leq i \leq n\}$ be the edge set of $H_n^{m_0}$. Define a total labeling f_{m_0} as follows:
For j odd

$$f_{m_0}(v_{i,j}) = \begin{cases} \frac{j-1}{2}n + 1, & \text{for } 1 \leq i \leq n;\ 1 \leq j \leq \frac{2m_0}{3} + 3, \\ \left\lceil \frac{(m_0+3)n+2}{3} \right\rceil, & \text{for } 1 \leq i \leq n;\ \frac{2m_0}{3} + 3 < j \leq m_0 + 1. \end{cases}$$

For j even

$$f_{m_0}(v_{i,j}) = \begin{cases} \frac{j-2}{2}n + i, & \text{for } 1 \leq i \leq n;\ 2 \leq j \leq \frac{2m_0}{3} + \frac{2}{n} + 2, \\ \left\lceil \frac{(m_0+3)n+2}{3} \right\rceil, & \text{for } 1 \leq i \leq n;\ \frac{2m_0}{3} + \frac{2}{n} + 2 < j \leq m_0 + 1, \end{cases}$$

$f_{m_0}(u_i) = \left\lceil \frac{(m_0+3)n+2}{3} \right\rceil$, $i = 1, \dots, n$,

$f_{m_0}(w) = \left\lceil \frac{(m_0+3)n+2}{3} \right\rceil$.

For $m_0 \le 6$ and $1 \le i \le n$, $1 \le j \le m_0$

$$f_{m_0}(v_{i,j}v_{i,j+1}) = 1.$$

For $m_0 > 6$

$$f_{m_0}(v_{i,j}v_{i,j+1}) = \begin{cases} 1, & \text{for } 1 \le i \le n;\ 1 \le j \le \frac{2m_0+6}{3}, \\ (j - \frac{2m_0+6}{3} - 1)n + i, & \text{for } 1 \le i \le n;\ \frac{2m_0+6}{3} < j \le m_0. \end{cases}$$

For $m_0 = 3$ and $1 \le i \le n$

$$f_{m_0}(v_{i,m_0+1}u_i) = 1.$$

For $m_0 > 3$ and $1 \le i \le n$

$$f_{m_0}(v_{i,m_0+1}u_i) = (\frac{m_0 - 6}{3})n + i.$$

$f_{m_0}(u_iu_{i+1(\text{mod } n)}) = (\frac{m_0-3}{3})n + i$, for $i = 1, \dots, n$,

$f_{m_0}(wu_i) = \frac{m_0}{3}n + i$, for $i = 1, \dots, n$.

It can be seen that the function f_{m_0} is a map from $V(H_n^{m_0}) \cup E(H_n^{m_0})$ into $\left\{1, 2, \dots, \left\lceil \frac{(m_0+3)n+2}{3} \right\rceil \right\}$. Thus, f_{m_0} is a total k_{m_0}-labeling with $k_{m_0} = \left\lceil \frac{(m_0+3)n+2}{3} \right\rceil$. Observe that

$$\begin{aligned} wt(v_{i,j}v_{i,j+1}) &= (j-1)n + 2 + i, i = 1, \dots, n; j = 1, \dots, m_0, \\ wt(v_{i,m_0+1}u_i) &= m_0n + 2 + i,\ i = 1, \dots, n, \\ wt(u_iu_{i+1(\text{mod n})}) &= (m_0+1)n + 2 + i, i = 1, \dots, n, \\ wt(wu_i) &= (m_0+2)n + 2 + i, i = 1, \dots, n. \end{aligned}$$

So the weights of edges of $H_n^{m_0}$ under the total k_{m_0}-labeling, f_{m_0}, form consecutive integers from 3 up to $(m_0+3)n+2$. It means that the weights of all edges are distinct. So, the labeling is an edge irregular total k_{m_0}-labeling with $k_{m_0} = \left\lceil \frac{(m_0+3)n+2}{3} \right\rceil$. Therefore $\text{tes}(H_n^{m_0}) = \left\lceil \frac{(m_0+3)n+2}{3} \right\rceil$, for $n \ge 3$. □

3. Conclusion

We have determined the total edge irregularity strength of generalized helm H_n^1, H_n^2, and H_n^m, and found that $\text{tes}(H_n^1) = \left\lceil \frac{4n+2}{3} \right\rceil$, $\text{tes}(H_n^2) = \left\lceil \frac{5n+2}{3} \right\rceil$ and $\text{tes}(H_n^m) = \left\lceil \frac{(m+3)n+2}{3} \right\rceil$, for $n \ge 3$ and $m \equiv 0 \pmod 3$.

Furthermore, we conclude this paper with the following conjecture for the direction of further research which is still in progress.

Conjecture 3.4. *The total edge irregularity strength of generalized helm H_n^m is* $tes(H_n^m) = \left\lceil \frac{(m+3)n+2}{3} \right\rceil$, *for* $n \geq 3$ *and* $m \geq 1$.

Acknowledgement

The authors wish to thank the referees for their valuable suggestions and references, which helped to improve the paper.

References

[1] A. Ahmad and M. Bača, Total edge irregularity strengths of the categorical product of two paths, *Arc Combin.*, (To appear).

[2] A. Ahmad, M. Bača, Y. Bashir and M. K. Siddiqui, Total edge irregularity strength of strong product of two paths, *Arc Combin.*, **106** (2012), 449–459.

[3] M. Bača, S. Jendrol', M. Miller and J. Ryan, On irregular total labeling, *Discrete Math.*, **307** (2007), 1378–1388.

[4] J.A. Gallian, A Dynamic Survey of Graph Labeling, *Electron. J. Combin.*, **19** (2012), # DS6.

[5] K.M.M. Haque, Irregular total labelings of Generalized Petersen Graphs, *Theory Comput Syst.*, **50** (2012), 537–544.

[6] D. Indriati, Widodo, I.E. Wijayanti and K.A. Sugeng, *Kekuatan Tak Regular Sisi Total pada Graf Helm dan Union Disjoinnya*, Presented in Konferensi Nasional Matematika XVI, Unpad, Jatinangor, Jawa Barat, $3^{rd} - 6^{th}$ July, 2012.

[7] J. Ivančo and S. Jendrol', Total edge irregularity strength of trees, *Discuss. Math. Graph Theory*, **26** (2006), 449–456.

[8] Nurdin, A.N.M. Salman and E.T. Baskoro, The total edge irregularity strengths of the corona product of paths with some graphs, *J. Combin. Math. Combin. Comput.*, **65** (2008), 163–175.

[9] M.K. Siddiqui, On edge irregularity strength of subdivision of star S_n, *International Journal of Mathematics and Soft Computing*, **2** (2012), 75–82.

[10] W.D. Wallis, *Magic Graphs*, Birkhauser, Boston, 2001.