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DNA graph characterization for line digraph of dicycle with one chord

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Abstract

Characterization of DNA graph gives important contribution in completing the computational step of Sequencing by Hybridization (SBH). Some graphs are already characterized as DNA graph using (α, k)-labeling. Dicycle and dipath are DNA graphs, while rooted trees and self adjoint digraphs are DNA graphs if and only if their maximum degree is not greater than four. In this paper we also use (α, k)-labeling to characterize line digraph of dicycle with one chord C_n^t as DNA graph and show that for $m, n \in \mathbb{N}, t = \left\lfloor \frac{n}{2} \right\rfloor, L^m(C_n^t)$ are DNA graphs.

Keywords: DNA graph, (α, k)-labeling.

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1. Introduction

Given a digraph $D = (V, A)$, every arc $a = ((u, v) \in A)$, u is called the tail of a and v is called the head of a. Line digraph of D, $L(D)$, is a digraph with vertex set $V(L(D)) = A(D)$ and an arc (x, y) exists in $A(L(D))$ if and only if the head of x is the tail of y in D.[1]

Line Digraph of $L(D)$ notated as $L(L(D))$ or simply $L^2(D)$, while line digraph of $L^m(D)$ notated as $L^{m+1}(D)$ for $m \in \mathbb{N}$.

In this paper we first reintroduce (α, k)-labeling and quasi (α, k)-labeling, and the relation between these two labelings as they had been introduced in [2]. The result will be used in section 2 to characterize $L^m(D)$, $t = \left\lfloor \frac{n}{2} \right\rfloor, m \in \mathbb{N}$ as DNA graph.

Given the integers $\alpha \geq 0$ and $k > 1$, we say that a digraph $D = (V, A)$ is (α, k)-labeled if it is possible to label each vertex x in D with a k-length label $(l_1(x), \ldots, l_k(x))$, which satisfy:

1. $l_i(x) \in \{1, \ldots, \alpha\}\ \forall x \in V, i = 1, \ldots, k$.
2. For any two vertices x and y, we have $l_i(x) \neq l_i(y), i = 1, \ldots, k$ if $x \neq y$.

3. $(x, y) \in A$ if and only if $l_i(x) = l_{i-1}(y), i = 2, \ldots, k$.

However, the third property makes it more complicated to find a (α, k)-labeling for a digraph. Hence to characterize a graph as a DNA graph, Li and Zhang in [2] defined a novel labeling of a graph, called quasi (α, k)-labeling.

Given the integers $\alpha \geq 0$ and $k > 1$, we say that a digraph $D = (V, A)$ is quasi (α, k)-labeled if it is possible to label each vertex x in D with a k-length label $(l_1^*(x), \ldots, l_k^*(x))$, which satisfy:

1. $l_i^*(x) \in \{1, \ldots, \alpha\}$ $\forall x \in V, i = 1, \ldots, k$.
2. For any two vertices x and y, we have $l_i^*(x) \neq l_i^*(y), i = 1, \ldots, k$ if $x \neq y$.
3. If $(x, y) \in A$ then $l_i^*(x) = l_{i-1}^*(y), i = 2, \ldots, k$.

Figure 1 shows a digraph D that is quasi (2,4)-labeled, but not (2,4)-labeled, for $l_i(p) = l_{i-1}(u), i = 2, 3, 4$, when (p, u) is not an arc in D. Lemma 1.1 gives the relation between two such labelings.

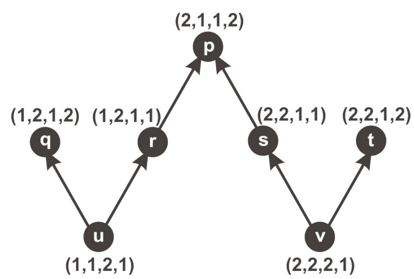
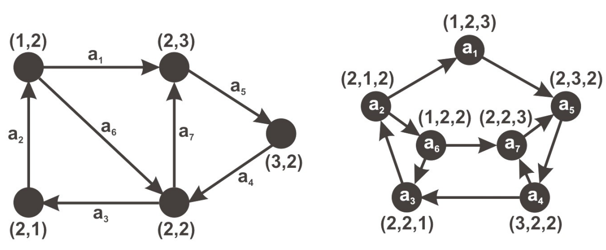


Figure 1. A graph with quasi (2,4)-labeling.

Lemma 1.1. *Given a digraph $D = (V, A)$, if D is quasi $(\alpha, k-1)$-labeled, then its line digraph, $L(D) = (V', A')$ is (α, k)-labeled.*



(a) (b)

Figure 2.(a) A quasi (3,2)-labeling of D and (b) a (3,3)-labeling of $L(D)$.

In [2] Li and Zhang introduced the definition of DNA graph as a digraph D with vertex set $V(D)$ and arc set $A(D)$ which is (α, k)-labeled for $\alpha \leq 4$ and $k > 1$. In this paper we will use this definition to characterize $L^m(D)$, $t = \lfloor \frac{n}{2} \rfloor, m \in \mathbb{N}$ as DNA graph.

2. Line Digraph of Dicycle with One Chord

Bača and Miller [1] defined dicycle with one chord C_n^t as a digraph constructed from dicycle C_n by adding an arc connecting two vertices which their distance to each other is t in C_n. In this paper, we characterize line digraph of dicycle with one chord $L(C_n^t)$ with $t = \lfloor \frac{n}{2} \rfloor$. Without loss of generality, we use $\left(v_{\lfloor \frac{n}{2} \rfloor}, v_n\right)$ as the extra arc. By Lemma 1.1, to find an (α, k)-labeling for $L\left(C_n^{\lfloor \frac{n}{2} \rfloor}\right)$, we need to find a quasi $(\alpha, k-1)$-labeling for $C_n^{\lfloor \frac{n}{2} \rfloor}$. Figure 3 shows that C_4^2 is quasi (3,2)-labeled.

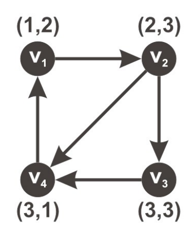


Figure 3. A quasi (3,2)-labeling for C_4^2.

Theorem 2.2. *For an even $n > 4$, $C_n^{\frac{n}{2}}$ is quasi $\left(4, \frac{n}{2}\right)$-labeled.*

Proof. Define l as labeling of $C_n^{\frac{n}{2}}$ with

$$l(v_1) = \left(1 \bmod 4, 2 \bmod 4, ..., \frac{n}{2} \bmod 4\right) \text{ and}$$
$$l(v_i) = \left(l_2(v_{i-1}), l_3(v_{i-1}), ..., l_{\frac{n}{2}-1}(v_{i-1}), l_{\frac{n}{2}}(v_{i-1}), l_{\frac{n}{2}}(v_i)\right), i = 2, ..., n$$

where

$$l_{\frac{n}{2}}(v_i) = \begin{cases} \left(\frac{n}{2}\right) \bmod 4 & \text{for} \quad i = 2 \\ (i-2) \bmod 4 & \text{for} \quad 3 \leq i < \frac{n}{2}+1 \\ (i-1) \bmod 4 & \text{for} \quad i = \frac{n}{2}+1 \\ \left(i - \left(\frac{n}{2}+1\right)\right) \bmod 4 & \text{for} \quad \frac{n}{2}+1 < i \leq n \end{cases}$$

For convenience, in this paper we take 4 mod 4 as 4 rather than 0.

It is obvious from the definition of l that it satisfies the first property of the quasi (α, k)-labeling. We will now show that $(x,y) \in A\left(C_n^{\frac{n}{2}}\right)$ implies $l_j(x) = l_{j-1}(y), j = 2, ..., \frac{n}{2}$ and any two distinct vertices in $C_n^{\frac{n}{2}}$ have different labels.

It is easy to see that $l_j(v_i) = l_{j-1}(v_{i-1}), i = 1, 2, ..., (n-1)$, so it is only necessary to show that $l_j(v_n) = l_{j-1}(v_1)$ and $l_j\left(\frac{n}{2}\right) = l_{j-1}(v_n), j = 2, ..., \frac{n}{2}$.

From the definition of l we have

$$\begin{aligned}
l(v_n) &= \left(l_1(v_n), l_2(v_n), ..., l_{\frac{n}{2}-1}(v_n), l_{\frac{n}{2}}(v_n)\right) \\
&= \left(l_2(v_{n-1}), l_3(v_{n-1}), ..., l_{\frac{n}{2}}(v_{n-1}), l_{\frac{n}{2}}(v_n)\right) \\
&= \left(l_3(v_{n-2}), l_4(v_{n-2}), ..., l_{\frac{n}{2}}(v_{n-2}), l_{\frac{n}{2}}(v_{n-1}), l_{\frac{n}{2}}(v_n)\right) \\
&\vdots \\
&= \left(l_{\frac{n}{2}}\left(v_{\frac{n}{2}+1}\right), l_{\frac{n}{2}}\left(v_{\frac{n}{2}+2}\right), ..., l_{\frac{n}{2}}(v_{n-2}), l_{\frac{n}{2}}(v_{n-1}), l_{\frac{n}{2}}(v_n)\right) \\
&= \left(\left(\frac{n}{2}\right) \bmod 4, 1 \bmod 4, 2 \bmod 4, ..., \left(\frac{n}{2}-2\right) \bmod 4, \left(\frac{n}{2}-1\right) \bmod 4\right) \\
&= \left(\left(\frac{n}{2} \bmod 4\right), l_1(v_1), l_2(v_1), ..., l_{\frac{n}{2}-1}(v_1)\right).
\end{aligned}$$

Thus $l_j(v_n) = l_{j-1}(v_1), j = 2, ..., \frac{n}{2}$.

Also

$$\begin{aligned}
l\left(v_{\frac{n}{2}}\right) &= \left(l_1\left(v_{\frac{n}{2}}\right), l_2\left(v_{\frac{n}{2}}\right), ..., l_{\frac{n}{2}-1}\left(v_{\frac{n}{2}}\right), l_{\frac{n}{2}}\left(v_{\frac{n}{2}}\right)\right) \\
&= \left(l_2\left(v_{\frac{n}{2}-1}\right), l_3\left(v_{\frac{n}{2}-1}\right), ..., l_{\frac{n}{2}-1}\left(v_{\frac{n}{2}-1}\right), l_{\frac{n}{2}}\left(v_{\frac{n}{2}}\right)\right) \\
&= \left(l_3\left(v_{\frac{n}{2}-2}\right), l_4\left(v_{\frac{n}{2}-2}\right), ..., l_{\frac{n}{2}}\left(v_{\frac{n}{2}-2}\right), l_{\frac{n}{2}}\left(v_{\frac{n}{2}-1}\right), l_{\frac{n}{2}}\left(v_{\frac{n}{2}}\right)\right) \\
&\vdots \\
&= \left(l_{\frac{n}{2}}(v_1), l_{\frac{n}{2}}(v_2), ..., l_{\frac{n}{2}}\left(v_{\frac{n}{2}-1}\right), l_{\frac{n}{2}}\left(v_{\frac{n}{2}}\right)\right) \\
&= \left(\left(\frac{n}{2}\right) \bmod 4, \left(\frac{n}{2}\right) \bmod 4, 1 \bmod 4, ..., \left(\frac{n}{2}-3\right) \bmod 4, \left(\frac{n}{2}-2\right) \bmod 4\right) \\
&= \left(\left(\frac{n}{2}\right) \bmod 4, \left(\frac{n}{2}\right) \bmod 4, l_1(v_1), l_2(v_1), ..., l_{\frac{n}{2}-2}(v_1)\right) \\
&= \left(\left(\frac{n}{2}\right) \bmod 4, l_1(v_n), l_2(v_n), l_3(v_n), ..., l_{\frac{n}{2}-1}(v_n)\right).
\end{aligned}$$

Thus $l_j\left(v_{\frac{n}{2}}\right)=l_{j-1}\left(v_n\right), j=2,...,\frac{n}{2}$.

We now show that any two distinct vertices in $C_n^{\frac{n}{2}}$ have different labels, through the following claims.

Claim 2.3. *For* $3\le i<\frac{n}{2}+1,\ l_1\left(v_i\right)=\left(l_{\frac{n}{2}}\left(v_i\right)+2\right)$ mod 4*.*

From the definition of l we have

$$l_{\frac{n}{2}}\left(v_i\right)=(i-2) \bmod 4 \text{ so that } l_{\frac{n}{2}}\left(v_i\right)+2=i \bmod 4=l_1\left(v_i\right).$$

Claim 2.4. *For* $\frac{n}{2}+1\le i<n,\ l_1\left(v_i\right)=l_{\frac{n}{2}}\left(v_i\right)$*.*

If $i=\frac{n}{2}+1$, from the definition of l we have

$$\begin{aligned}l_{\frac{n}{2}}\left(v_i\right)=(i-1) \bmod 4=\left(\frac{n}{2}\right) \bmod 4=l_{\frac{n}{2}}\left(v_2\right)=l_{\frac{n}{2}-1}\left(v_3\right)=\dots\\=l_3\left(v_{\frac{n}{2}-1}\right)=l_2\left(v_{\frac{n}{2}}\right)=l_1\left(v_{\frac{n}{2}+1}\right)=l_1\left(v_1\right).\end{aligned}$$

If $\frac{n}{2}+1<i<n$, note that for $3\le i<\frac{n}{2}+1$, $\left(i-\left(\frac{n}{2}+1\right)\right)$ mod 4 is equal to $(i-1)$ mod 4, which implies $l_{\frac{n}{2}}\left(v_{\frac{n}{2}+x}\right)=l_{\frac{n}{2}}\left(v_{x+1}\right), i<x<\frac{n}{2}$. Thus we have

$$\begin{aligned}l_1\left(v_i\right)=l_2\left(v_{i-1}\right)=\dots=l_{\frac{n}{2}}\left(v_{i-\left(\frac{n}{2}-1\right)}\right), &\qquad \frac{n}{2}+1<i<n\\=l_{\frac{n}{2}}\left(v_{x+1}\right)=l_{\frac{n}{2}}\left(v_{\frac{n}{2}+x}\right), &\qquad 1<x<\frac{n}{2}\\=l_{\frac{n}{2}}\left(v_i\right), &\qquad \frac{n}{2}+1<i<n\end{aligned}$$

Claim 2.5. *For* $i=n,\ l_1\left(v_i\right)=\left(l_{\frac{n}{2}}\left(v_i\right)+1\right)$ mod 4*.*

$l_{\frac{n}{2}}\left(v_n\right)=\left(n-\left(\frac{n}{2}+1\right)\right)$ mod 4 $=\left(\frac{n}{2}-1\right)$ mod 4 so that

$$\begin{aligned}\left(l_{\frac{n}{2}}\left(v_n\right)+1\right) \bmod 4=\left(\frac{n}{2}\right) \bmod 4=l_{\frac{n}{2}}\left(v_2\right)=l_{\frac{n}{2}-1}\left(v_3\right)=\dots=l_1\left(v_{\frac{n}{2}+1}\right)\\=l_{\frac{n}{2}}\left(v_{\frac{n}{2}+1}\right)=l_{\frac{n}{2}-1}\left(v_{\frac{n}{2}+2}\right)=\dots=l_1\left(v_n\right).\end{aligned}$$

Claim 2.6. *For* $3\le i<\frac{n}{2}+1$*, when* $j=\frac{n}{2}+1-i$*, either there is exactly one pair of twins* $l_j\left(v_i\right)=l_{j+1}\left(v_i\right)$ *or there is exactly one group of triplets* $l_j\left(v_i\right)=l_{j+1}\left(v_i\right)=l_{j+2}\left(v_i\right)$*. For other values of* j*,* $l_{j+1}\left(v_i\right)=\left(l_j\left(v_j\right)+1\right)$ mod 4*.*

If n is a multiple of 10, $l_{\frac{n}{2}}(v_2) = 5 \bmod 4 = 1 \bmod 4$. By the definition of l, $l_{\frac{n}{2}}(v_3) = (3-2) \bmod 4 = 1 \bmod 4$. Thus for $i = 3$, $j = \frac{n}{2} + 1 - i$, $l_j(v_i) = l_{j+1}(v_i) = l_{j+2}(v_i)$ is a triplet. If n is not a multiple of 10, $l_{\frac{n}{2}}(v_2) \neq 1 \bmod 4$. Hence for $j = \frac{n}{2} + 1 - i$, $l_j(v_i) = l_{j+1}(v_i)$ is a pair of twins. For other values of j this lemma holds by the definition of l.

Claim 2.7. *For $\frac{n}{2} + 1 \leq i < n$, if $j = n - i$, then $l_{j+1}(v_i) = (l_j(v_i) + 2) \bmod 4$ and if $j + 2 < \frac{n}{2}$, then $l_{j+2}(v_i) = 1$. For other values of j, $l_{j+1}(v_i) = (l_j(v_i) + 1) \bmod 4$.*

By the definition of l,

$$\begin{aligned} l_{\frac{n}{2}}\left(v_{\frac{n}{2}+1}\right) &= \left(\frac{n}{2} + 1 - 1\right) \bmod 4 = \left(\frac{n}{2}\right) \bmod 4 = \left(\frac{n}{2} + 2 - 2\right) \bmod 4 \\ &= \left(l_{\frac{n}{2}}\left(v_{\frac{n}{2}}\right) + 2\right) \bmod 4 = \left(l_{\frac{n}{2}-1}\left(v_{\frac{n}{2}+1}\right) + 2\right) \bmod 4 \end{aligned}$$

So when $j = n - 1$, $i = \frac{n}{2} + 1$,

$$\begin{aligned} l_{n-\frac{n}{2}}\left(v_{\frac{n}{2}+1}\right) = l_{\frac{n}{2}}\left(v_{\frac{n}{2}+1}\right) &= \left(l_{\frac{n}{2}-1}\left(v_{\frac{n}{2}+1}\right) + 2\right) \bmod 4 \\ &= \left(l_{n-\left(\frac{n}{2}+1\right)}\left(v_{\frac{n}{2}+1}\right) + 2\right) \bmod 4. \end{aligned}$$

Claim 2.3 to Claim 2.7 are used to prove that any pair of distinct vertices in $C_n^{\frac{n}{2}}$ have different labels. Consider two arbitrary vertices v_p and v_q in $C_n^{\frac{n}{2}}$. Table 1 shows how any two distinct vertices in $C_n^{\frac{n}{2}}$ have different labels.

p	q	Property 2 holds based on
1	2	Definition of l
	$3 \leq q < \frac{n}{2} + 1$	Claim 2.6
	$\frac{n}{2} + 1 \leq q < n$	Claim 2.7
	n	Definition of l and property 3
2	$3 \leq q < \frac{n}{2} + 1$	Claim 2.6
	$\frac{n}{2} + 1 \leq q < n$	Claim 2.7
	n	Definition of l and property 3
$3 \leq p < \frac{n}{2} + 1$	$3 \leq q < \frac{n}{2} + 1$	Claim 2.6
	$\frac{n}{2} + 1 \leq q < n$	Claim 2.3 and Claim 2.4
	n	Claim 2.3 and Claim 2.5
$\frac{n}{2} + 1 \leq p < n$	$\frac{n}{2} + 1 \leq q < n$	Claim 2.7
	n	Claim 2.4 and Claim 2.5

Table 1

Hence l is a quasi $\left(4, \frac{n}{2}\right)$-labeling for $C_n^{\frac{n}{2}}$. □

Theorem 2.8. *For an odd $n > 4$, $C_n^{\lfloor \frac{n}{2} \rfloor}$ is quasi $\left(4, \lceil \frac{n}{2} \rceil\right)$-labeled.*

Proof. Define the labeling l^* as follows.

$$l^*(v_1) = \left(4, 2 \bmod 4, ..., \left\lceil \frac{n}{2} \right\rceil \bmod 4\right) \text{ and}$$
$$l^*(v_i) = \left(l_2^*(v_{i-1}), l_3^*(v_{i-1}), ..., l^*_{\lceil \frac{n}{2} \rceil}(v_{i-1}), l^*_{\lceil \frac{n}{2} \rceil}(v_i)\right), i = 2, ..., n$$

where

$$l^*_{\lceil \frac{n}{2} \rceil}(v_i) = \begin{cases} 4 & \text{for} \quad i = 2 \\ (i-1) \bmod 4 & \text{for} \quad 3 \le i < \lceil \frac{n}{2} \rceil + 1 \\ i \bmod 4 & \text{for} \quad i = \lceil \frac{n}{2} \rceil \\ 4 & \text{for} \quad i = \lceil \frac{n}{2} \rceil + 1 \\ \left(i - \lceil \frac{n}{2} \rceil\right) \bmod 4 & \text{for} \quad \lceil \frac{n}{2} \rceil + 1 < i \le n \end{cases}$$

It is obvious from the definition of l^* that it satisfies the first property of the quasi (α, k)-labeling. We will now show that $(x, y) \in A\left(C_n^{\lfloor \frac{n}{2} \rfloor}\right)$ implies $l_j^*(x) = l_{j-1}^*(y), j = 2, ..., \lceil \frac{n}{2} \rceil$ and any two distinct vertices in $C_n^{\lfloor \frac{n}{2} \rfloor}$ have different labels.

It is easy to see that $l_j^*(v_i) = l_{j-1}^*(v_{i-1}), i = 1, 2, ..., (n-1)$, so it is only necessary to show that $l_j^*(v_n) = l_{j-1}^*(1)$ and $l_j^*\left(v_{\lfloor \frac{n}{2} \rfloor}\right) = l_{j-1}^*(v_n), j = 2, ..., \lceil \frac{n}{2} \rceil$.

From the definition of l^*, we have

$$\begin{aligned} l^*(v_n) &= \left(l_1^*(v_n), l_2^*(v_n), ..., l^*_{\lceil \frac{n}{2} \rceil - 1}(v_n), l^*_{\lceil \frac{n}{2} \rceil}(v_n)\right) \\ &= \left(l_2^*(v_{n-1}), l_3^*(v_{n-1}), ..., l^*_{\lceil \frac{n}{2} \rceil}(v_{n-1}), l^*_{\lceil \frac{n}{2} \rceil}(v_n)\right) \\ &= \left(l_3^*(v_{n-2}), l_4^*(v_{n-2}), ..., l^*_{\lceil \frac{n}{2} \rceil}(v_{n-2}), l^*_{\lceil \frac{n}{2} \rceil}(v_{n-1}), l^*_{\lceil \frac{n}{2} \rceil}(v_n)\right) \\ &\vdots \\ &= \left(l^*_{\lceil \frac{n}{2} \rceil}\left(v_{\lceil \frac{n}{2} \rceil}\right), l^*_{\lceil \frac{n}{2} \rceil}\left(v_{\lceil \frac{n}{2} \rceil + 1}\right), ..., l^*_{\lceil \frac{n}{2} \rceil}(v_{n-2}), l^*_{\lceil \frac{n}{2} \rceil}(v_{n-1}), l^*_{\lceil \frac{n}{2} \rceil}(v_n)\right) \\ &= \left(\left\lceil \frac{n}{2} \right\rceil \bmod 4, 4, \left(\left\lceil \frac{n}{2} \right\rceil + 2 - \left\lceil \frac{n}{2} \right\rceil\right) \bmod 4, ..., \left(n - \left\lceil \frac{n}{2} \right\rceil\right) \bmod 4\right). \end{aligned}$$

We will now show that any two distinct vertices in $C_n^{\lfloor \frac{n}{2} \rfloor}$ have different labels, through the following claims.

Claim 2.9. *For* $3 \leq i < \lceil \frac{n}{2} \rceil,\ l_1^*(v_i) = \left(l_{\lceil \frac{n}{2} \rceil}(v_i) + 1 \right) \bmod 4.$

From the definition of l^*,

$$l^*_{\lceil \frac{n}{2} \rceil}(v_i) = (i-1) \bmod 4 \text{ so that } \left(l^*_{\lceil \frac{n}{2} \rceil}(v_i) + 1 \right) \bmod 4 = i \bmod 4 = l_1^*(v_i).$$

Claim 2.10. *For* $\lceil \frac{n}{2} \rceil \leq i < n,\ l_1^*(v_i) = l^*_{\lceil \frac{n}{2} \rceil}(v_i).$

If $i = \lceil \frac{n}{2} \rceil$,

$$l^*_{\lceil \frac{n}{2} \rceil}(v_i) = \left\lceil \frac{n}{2} \right\rceil \bmod 4 = l^*_{\lceil \frac{n}{2} \rceil}(v_1) = l^*_{\lceil \frac{n}{2} \rceil - 1}(v_2) = l^*_{\lceil \frac{n}{2} \rceil - 2}(v_3) = \ldots = l_1^*\left(v_{\lceil \frac{n}{2} \rceil} \right).$$

If $i = \lceil \frac{n}{2} \rceil + 1$,

$$l^*_{\lceil \frac{n}{2} \rceil}(v_i) = 4 \bmod 4 = l^*_{\lceil \frac{n}{2} \rceil}(v_2) = l^*_{\lceil \frac{n}{2} \rceil - 1}(v_3) = l^*_{\lceil \frac{n}{2} \rceil - 2}(v_4) = \ldots = l_1^*\left(v_{\lceil \frac{n}{2} \rceil + 1} \right).$$

If $i = \lceil \frac{n}{2} \rceil + 1 < i < n$ note that $\left(i - \lceil \frac{n}{2} \rceil \right) \bmod 4$ is equal to $(i-1) \bmod 4$ when $3 \leq i < \lceil \frac{n}{2} \rceil$, so that

$$l^*_{\lceil \frac{n}{2} \rceil}\left(v_{\lceil \frac{n}{2} \rceil + x} \right) = l^*_{\lceil \frac{n}{2} \rceil}(v_{x+1}) \text{ for } 1 < x < \left\lfloor \frac{n}{2} \right\rfloor.$$

Thus we have

$$\begin{aligned} l_1^*(v_i) = l_2^*(v_{i-1}) = \ldots = l^*_{\lceil \frac{n}{2} \rceil}\left(v_{i - \lceil \frac{n}{2} \rceil} \right), && \left\lceil \frac{n}{2} \right\rceil + 1 < i < n \\ = l^*_{\lceil \frac{n}{2} \rceil}(v_{x+1}) = l^*_{\lceil \frac{n}{2} \rceil}\left(v_{\lceil \frac{n}{2} \rceil + x} \right), && 1 < x < \left\lfloor \frac{n}{2} \right\rfloor \\ = l^*_{\lceil \frac{n}{2} \rceil}(v_i), && \left\lceil \frac{n}{2} \right\rceil + 1 < i < n \end{aligned}$$

Claim 2.11. *For* $i = n,\ l_1^*(v_i) = \left(l^*_{\lceil \frac{n}{2} \rceil}(v_i) + 1 \right) \bmod 4$

$l^*_{\lceil \frac{n}{2} \rceil}(v_n) = \left(n - \lceil \frac{n}{2} \rceil \right) \bmod 4 = \left(\lceil \frac{n}{2} \rceil - 1 \right) \bmod 4$ so that

$$\begin{aligned} \left(l^*_{\lceil \frac{n}{2} \rceil}(v_n) + 1 \right) \bmod 4 = \left\lceil \frac{n}{2} \right\rceil \bmod 4 = l^*_{\lceil \frac{n}{2} \rceil}\left(v_{\lceil \frac{n}{2} \rceil} \right) = l^*_{\lceil \frac{n}{2} \rceil - 1}\left(v_{\lfloor \frac{n}{2} \rfloor + 1} \right) \\ = l^*_{\lceil \frac{n}{2} \rceil - 2}\left(v_{\lfloor \frac{n}{2} \rfloor + 2} \right) = \ldots = l_1^*(v_n) \end{aligned}$$

Claim 2.12. *For $1 \le i \le n, i \ne \lceil \frac{n}{2} \rceil + 1$, there is exactly one sequence*

$$\left(l^*_{(j+x) \bmod \lceil \frac{n}{2} \rceil}(v_i)\right) = \left(\left\lceil \frac{n}{2} \right\rceil \bmod 4, 4, 2\right),\ x = 1, 2$$

and for $i = \lceil \frac{n}{2} \rceil + 1$, there is exactly one sequence

$$\left(l^*_{(j+x) \bmod \lceil \frac{n}{2} \rceil}(v_i)\right) = \left(\left\lceil \frac{n}{2} \right\rceil \bmod 4, 4, 4, 2\right),\ x = 1, 2, 3$$

that is, when

$$j = \begin{cases} \lceil \frac{n}{2} \rceil & \text{for} \quad i = 1 \\ \lceil \frac{n}{2} \rceil - 1 & \text{for} \quad i = 2 \\ \lceil \frac{n}{2} \rceil - i + 1 & \text{for} \quad 3 \le i < \lceil \frac{n}{2} \rceil + 1 \\ 1 & \text{for} \quad i = \lceil \frac{n}{2} \rceil \\ n - i + 1 & \text{for} \quad \lceil \frac{n}{2} \rceil + 1 < i \le n \end{cases}$$

while for other values of j, $l^*_{j+1}(v_j) = \left(l^*_j(v_i)\right) \bmod 4$.

For $1 \le i \le 3$, this claim holds from the definition of l^*.
For $3 < i < \lceil \frac{n}{2} \rceil + 1$, this Claim holds by property 3 of quasi labeling of l^*.
For $i = \lceil \frac{n}{2} \rceil + 1$,

$$l^*_{(j+x) \bmod \lceil \frac{n}{2} \rceil}(v_i) = l^*_{\left(n - \left(\lceil \frac{n}{2} \rceil + 1\right) + 1 + 1\right) \bmod \lceil \frac{n}{2} \rceil}\left(v_{\lceil \frac{n}{2} \rceil + 1}\right) = l^*_{\lceil \frac{n}{2} \rceil}\left(v_{\lceil \frac{n}{2} \rceil + 1}\right) = 4,\ x = 1$$

while for $x = 0, 2, 3$ the equation holds by property 3 of quasi labeling of l^*.

For $\lceil \frac{n}{2} \rceil + 1 < i \le n$, this Claim holds by property 3 of quasi labeling of l^*.

Claim 2.9 to Claim 2.12 are used to prove that any pair of distinct vertices in $C_n^{\lfloor \frac{n}{2} \rfloor}$ have different labels. Consider two arbitrary vertices v_p and v_q in $C_n^{\lfloor \frac{n}{2} \rfloor}$. Table 2 shows how any two distinct vertices in $C_n^{\lfloor \frac{n}{2} \rfloor}$ have different labels.

p	q	Property 2 holds based on
1	2	Definition of l^*
	$3 \leq q < \frac{n}{2}+1$	Claim 2.12
	$\lceil \frac{n}{2} \rceil \leq q < n$	Claim 2.12
	n	Definition of l^* and property 3
2	$3 \leq q < \lceil \frac{n}{2} \rceil$	Claim 2.12
	$\lceil \frac{n}{2} \rceil \leq q < n$	Claim 2.12
	n	Definition of l^* and property 3
$3 \leq p < \lceil \frac{n}{2} \rceil$	$3 \leq q < \frac{n}{2}+1$	Claim 2.12
	$\lceil \frac{n}{2} \rceil \leq q < n$	Claim 2.9 and Claim 2.10
	n	Claim 2.9 and Claim 2.11
$\lceil \frac{n}{2} \rceil \leq p < n$	$\lceil \frac{n}{2} \rceil \leq q < n$	Claim 2.10
	n	Claim 2.10 and Claim 2.11

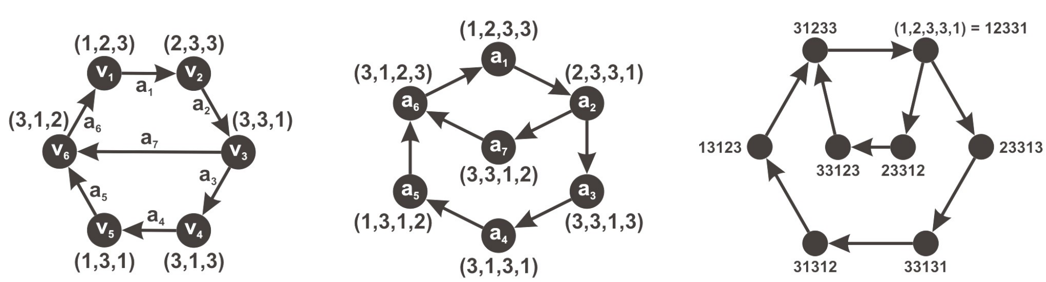
Table 2

Hence l^* is a quasi $\left(4, \lceil \frac{n}{2} \rceil\right)$-labeling for $C_n^{\lfloor \frac{n}{2} \rfloor}$. □

Combining Lemma 1.1, Theorem 2.2, Theorem 2.8 and the definition of DNA graph, we have the following.

Corollary 2.13. *For $n \geq 4$ and $m \geq 1$, $L^m\left(C_n^{\lfloor \frac{n}{2} \rfloor}\right)$ are DNA graphs.*

Figure 4 shows quasi $(3,3)$-labeling for C_6^3 and (α, k)-labeling for its first two line digraphs, which are DNA graphs.



(a) (b) (c)

Figure 4.(a) Quasi (3,3)-labeling of C_6^3, (b) (3,4)-labeling of $L\left(C_6^3\right)$ and (c) (3,5)-labeling of $L^2\left(C_6^3\right)$

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References

[1] M. Bača and M. Miller, *Super Edge-Antimagic Graphs*, Brown Walker Press, 2008.

[2] X. Li and H. Zhang, Characterization for Some Types of DNA Graphs, *J. Math. Chem.*, **42**(2) (2006), 65–79.