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Further results on cycle-supermagic labeling

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Abstract

For a graph $G(V, E)$, an edge-covering of G is a family of distinct subgraphs $H_1, \dots H_k$ such that any edge of E belongs to at least one of the subgraphs $H_i, 1 \leq i \leq k$. If every H_i is isomorphic to a given graph H, then G admits an H-covering. Graph G is said to be H-magic if G has an H-covering and there is a total labeling $f : V \cup E \to \{1, 2, \dots, |V|+|E|\}$ such that for each subgraph $H' = (V', E')$ of G isomorphic to H, $\sum_{v \in V'} f(v) + \sum_{e \in E'} f(e)$ is a fixed constant. Furthermore, if $f(V) = \{1, 2, \dots, |V|\}$ then G is called H-supermagic. The sum of all vertex and edge labels on H, under a labeling f, is denoted by $\sum f(H)$.

In this paper we study H-supermagic labeling for some classes of graphs such as a Jahangir graph, a wheel graph for even n, and a complete bipartie graph $K_{m,n}$ for $m = 2$.

Keywords: H-supermagic labeling, Jahangir graph, wheel graph, complete bipartite graph.

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1. Introduction

We consider only finite and simple graphs. The vertex-set of a graph G is denoted by $V(G)$ and the edge-set by $E(G)$. An *edge-covering* of G is a family of distinct subgraphs $H_1, \dots H_k$ such that any edge of E belongs to at least one of the subgraphs $H_i, 1 \leq i \leq k$. If every H_i is isomorphic to a given graph H, then G admits an *H-covering*. A *total H-magic labeling* of G is an injection $f : V \cup E \to \{1, 2, \dots |V| + |E|\}$ such that for each subgraph $H' = (V', E')$ of G isomorphic to H, we have $\sum_{v \in V'} f(v) + \sum_{e \in E'} f(e)$ is constant. Furthermore, if $f(V) = \{1, 2, \dots |V|\}$ then G is called *H-supermagic*.

The H-supermagic labeling was first introduced by Gutiérrez and Lladó [2] in 2005, in which H-supermagic labeling of stars, complete bipartite graphs, paths and cycles were found. In [4], Lladó and Morragas studied C_n-supermagic labeling of some graphs, such as wheels, windmills, prisms and books. They proved that those graphs mentioned are C_h-supermagic for some h. In [5], Maryati et al. proved that some classes of trees such as subdivision of stars, shrubs and banana trees are P_h-supermagic. Furthermore, cycles-supermagic labeling of chain graphs kC_n-snake, triangle ladders TL_n, grids $P_m \Box P_n$ for $n =$ 2,3,4,5, also fans F_n and books B_n are H-magic in Ngurah et al. [8]. For certain schakles and amalgamations of a connected graph, Maryati et al. [6] had shown the result on the same topic, and a path-amalgamation of isomorphic graphs had also been proved by Salman and Maryati (see [10]). Also, Jeyanthi and Selvagopal in [3] proved that one point union, garland graph and linear garland admit H-supermagic. Recent results of Roswitha and Baskoro in [9] showed that some trees, such as a double star, a caterpillar, a firecracker and a banana tree admit star-supermagic labeling.

2. k-balanced multisets

In this section, we use a technique of partitioning a multi-set that was introduced by Maryati et al. [6]. The notation $[a, b]$ is to mean $\{x \in N | a \leq x \leq b\}$ and the notation $\sum X$ is for $\sum_{x \in X} x$. Here we also define that $\{a\} \uplus \{a, b\} = \{a, a, b\}$.

Maryati et al. [6, 7] defined that a multi-set is a set that allows the existence of the same elements in it. Let $k \in N$ and Y be a multi-set that contains positive integers. The multi-set Y is said to be *k-balanced* if there exist k subsets of Y, namely $Y_1, Y_2, ..., Y_k$ such that for every $i \in [1, k], |Y_i| = \frac{|Y|}{k}, \sum Y_i = \frac{\sum Y}{k} \in N$, and $\biguplus_{i=1}^{k} Y_i = Y$. If those hold for every $i \in [1, k]$ then Y_i is called a *balanced subset* of Y. The following two lemmas have been proved by Maryati et al. [6, 7].

Lemma 2.1. [6] *Let x, y and z be nonnegative integers and $k \geq 3$ be an odd integer. Then the multi-set $Y = [x + 1, x + k] \uplus [y + 1, y + k] \uplus [z + 1, z + k]$ is k-balanced.*

Lemma 2.2. [7] *Let x, y and k be integers such that $1 \leq x < y$ and $k > 1$. If $X = [x, y]$ and X is a multiple of $2k$ then X is k-balanced.*

We have the following Lemmas.

Lemma 2.3. *Let x and k be nonnegative integers. Let $X = [x + 1, x + k]$ with $|X| = k$ and $Y = [x + k + 1, x + 2k]$ where $|Y| = k$. Then, the multiset $K = X \uplus Y$ is k-balanced for $j \in [1, k]$.*

Proof. Let $K = \{\{a_j, b_j\} | 1 \leq j \leq k\}$, where

$$\begin{aligned} a_j &= x + j, & j \in [1, k] \\ b_j &= x + 2k + 1 - j, & j \in [1, k]. \end{aligned}$$

Next, we define the sets

$$\begin{aligned} A &= \{a_i | 1 \leq j \leq k\} = [x+1, x+k] \\ B &= \{b_i | 1 \leq j \leq k\} = [x+k+1, x+2k]. \end{aligned}$$

Since $A \biguplus B = K$ we have $\biguplus_{j=1}^{k} K_j = K$ and K_j are 2-sets. Then $\sum K_j = 2x + 2k + 1$ for $1 \leq j \leq k$, and K is k-balanced. □

Lemma 2.4. *Let x, y, z and k be positive integers. Then the multiset $Y = [x+2, x+k+1] \uplus [y+1, y+k] \uplus [z+1, z+k]$ is k-balanced for odd $k > 2$.*

Proof. For odd $k > 2$ and every $i \in [1, k]$, we define the multisets $Y_i = \{a_i, b_i, c_i\}$, where

$$a_i = x + i + 1 \qquad \text{for} \quad i \in [1, k];$$

$$b_i = \begin{cases} y + \lceil \frac{k}{2} \rceil + i - 1 & \text{for } i \in [1, \lceil \frac{k}{2} \rceil]; \\ y - \lceil \frac{k}{2} \rceil + i & \text{for } i \in [\lceil \frac{k}{2} \rceil + 1, k]; \end{cases}$$

$$c_i = \begin{cases} z + k - 2i + 2 & \text{for } i \in [1, \lceil \frac{k}{2} \rceil]; \\ z + 2(k-i) + 2\lceil \frac{k}{2} \rceil - 2 & \text{for } i \in [\lceil \frac{k}{2} \rceil + 1, k]. \end{cases}$$

Furthermore, we define

$$\begin{aligned} A &= \{a_i | 1 \leq i \leq k\} = [x+2, x+k+1]; \\ B &= \{b_i | 1 \leq i \leq k\} = [y+1, y+k]; \\ C &= \{c_i | 1 \leq i \leq k\} = [z+1, z+k]. \end{aligned}$$

If $A \uplus B \uplus C = Y$ and $\biguplus_{i=1}^{k} Y_i = Y$ then for every $i \in [1, k]$ we obtain $|Y_i| = 3$ and $\sum Y_i = x + y + z + k + \lceil \frac{k}{2} \rceil + 2$. Hence Y is k-balanced. □

Lemma 2.5. *Let x, y, z and k be positive integers. Then the multiset $Y = [x+1, x+k+1] \setminus \{k\} \uplus [y+1, y+k] \uplus [z+1, z+k]$ is k-balanced for even $k \geq 4$.*

Proof. For even $k \geq 4$ and every $i \in [1, k]$, we define the multisets $Y_i = \{a_i, b_i, c_i\}$, where

$$a_i = \begin{cases} x + \lfloor \frac{i}{2} \rfloor + 1 & \text{for } i \text{ odd}; \\ x + (\frac{k+i}{2}) + 1 & \text{for } i \text{ even}; \end{cases}$$

$$b_i = \begin{cases} y + \frac{k}{2} + \lfloor \frac{i}{2} \rfloor + 1 & \text{for } i \text{ odd}; \\ y + \frac{i}{2} & \text{for } i \text{ even}; \end{cases}$$

$$c_i = z + k - i + 1 \qquad \text{for} \quad i \in [1, k].$$

Next, we define

$$A = \{a_i | 1 \le i \le k\} = [x+1, x+k+1];$$
$$B = \{b_i | 1 \le i \le k\} = [y+1, y+k];$$
$$C = \{c_i | 1 \le i \le k\} = [z+1, z+k].$$

If $A \uplus B \uplus C = Y$ and $\biguplus_{i=1}^{k} Y_i = Y$, then for every $i \in [1, k]$ we obtain $|Y_i| = 3$ so that $\sum Y_i = x + y + z + \frac{3k}{2} + 2$. We conclude that Y is k-balanced. □

3. Generalized Jahangir Graph

A *generalized Jahangir graph* $J_{k,s}$ is a graph on $ks+1$ vertices consisting of a cycle C_{ks} and one additional vertex that is adjacent to k vertices of C_{ks} at distance s to each other on C_{ks} (Gallian [1]). If $s = 2$ then the graph is called a *Jahangir graph* or *gear graph.* Lladó and Moragas in [4] showed that the wheel W_n for $n \ge 5$ odd, is C_3-supermagic. In this section we prove that a generalized Jahangir graph $J_{k,s}$ is C_{s+2}-supermagic for any odd k. As corollary, a gear graph $J_{k,2}$ is C_4-supermagic for any odd k.

The following theorem shows that a generalized Jahangir graph $J_{k,s}$ is C_{s+2}-supermagic for any odd k.

Theorem 3.1. *Any generalized Jahangir graph $J_{k,s}$ is C_{s+2}-supermagic for odd k.*

Proof. Let G be a Jahangir graph $J_{k,s}$ with $|V(G)| = ks+1$, with k vertices $\{v_1, v_2, ...v_k\}$ adjacent to one vertex v_0 and additional s vertices between v_1 and v_2, s vertices between v_2 and v_3, etc., and $|E(G)| = ks+k$. Let $Z = [1, 2ks+k+1]$. Partition Z into 4 sets: $X = [2, k+1], Y = [k+2, ks+1], K = [ks+2, ks+k+1]$ and $L = [ks+k+2, 2ks+k+1]$.

Let H be any C_{s+2} subgraph of $J_{k,s}$, with $V(H) = \{v_0, v_1, v_{k+1}, ..., v_2, v_0\}, \{v_0, v_2, v_{k+s+1}, ..., v_3, v_0\}, \; ... \; ,\{v_0, v_k, v_{k+2s+1}, ..., v_1, v_0\}$. Here we have kC_{s+2}-subgraphs of $J_{k,s}$. Furthermore, we define a total labeling $f(H)$ as follows. First, set $f(v_0) = 1$, the central vertex of the generalized Jahangir graph, then put elements of X as labels of k vertices adjacent to v_0, i.e. $f(v_1) = 2, ..., f(v_k) = k+1$. Label the remaining vertices by elements of Y as follows. Assume that $k \ge 3, k$ odd, and for every $i \in [1, k]$ we define a multiset $Y_i = \{a_i, b_i, c_i\}$ where

$$
\begin{aligned}
a_i &= x + i \\
b_i &= \begin{cases} y + \lceil \frac{k}{2} \rceil + i & \text{for } i \in [1, \lfloor \frac{k}{2} \rfloor] \\ y - \lfloor \frac{k}{2} \rfloor + i & \text{for } i \in [\lceil \frac{k}{2} \rceil, k]. \end{cases} \\
c_i &= \begin{cases} z + k + 1 - 2i & \text{for } i \in [1, \lfloor \frac{k}{2} \rfloor] \\ z + k + 2\lceil \frac{k}{2} \rceil - 2i & \text{for } i \in [\lceil \frac{k}{2} \rceil, k]. \end{cases}
\end{aligned}
$$

Furthemore, we define the sets

$$\begin{array}{ll} A & = \{a_i | 1 \le i \le k\} = [x+1, x+k] \\ B & = \{b_i | 1 \le i \le k\} = [y+1, y+k] \\ C & = \{c_i | 1 \le i \le k\} = [z+1, z+k]. \end{array}$$

It is obvious that $A \biguplus B \biguplus C = Y$ and $\biguplus_{i=1}^{k} Y_i = Y$. Then by Lemma 2.1, for $i \in [1, k]$, $|Y| = 3$ and $\sum Y_i = x + y + z + \lceil \frac{3k}{2} \rceil + 1$. If we put $x = k+1, y = 2k+1$ and $z = 3k+1$, then we have $\sum Y_i = 6k + \lceil \frac{3k}{2} \rceil + 4$. Therefore Y is k-balanced.

We now label all edges incident to v_0 by the elements of K as follows, $f(v_0v_1) = ks+k+1, f(v_0v_2) = ks+k, ..., f(v_0v_k) = ks+2$. Let $M = X \biguplus K$. For every $i \in [1, k]$, we partition M_i into some 2-sets as in Lemma 2.3 such that $\sum M_i = ks + k + 3$.

Next, all edges on the outer cycle of C_{ks} are labeled by the elements of L. Since $|L|$ is the multiple of $2k$, by Lemma 2.2, we define $L_i = \{a_j^i | 1 \le j \le \frac{|L|}{k}\}$ where

$$a_i^j \quad = \begin{cases} x - 1 + k(j-1) + i & \text{for odd } j \\ x + kj - i & \text{for even } j. \end{cases}$$

For every $i \in [1, k]$, it holds that $|L_i| = \frac{|L|}{k}$. According to Lemma 2.2, L is k-balanced, with $\sum L_i = \frac{|L|}{2k}(x+y)$. If $x = ks + k + 2$ and $y = 2ks + k + 1$, then $\sum L_i = \frac{|L|}{2k}(3ks + 2k + 3)$. We conclude that for every $i \in [1, k]$, we obtain $f(H_i) = 2(\sum M_i) + 1 + \sum Y_i + \sum L_i$, which is constant. Hence, the generalized Jahangir graph $J_{k,s}$ admits an C_{s+2}-supermagic labeling. □

Corollary 3.2. *The Jahangir graph $J_{k,2}$ or gear graph is C_4-supermagic for k odd.*

Figure 1 and Figure 2 illustrate an example of C_6-supermagic labeling on a generalized Jahangir graph $J_{5,4}$ and an example of C_4-supermagic labeling on a gear graph or a Jahangir graph $J_{5,2}$ respectively.

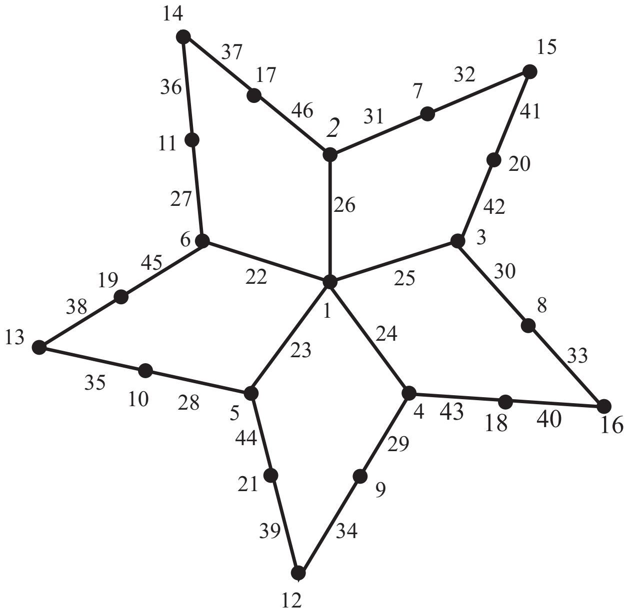

Figure 1: C_6-supermagic labeling on a generalized Jahangir graph $J_{5,4}$

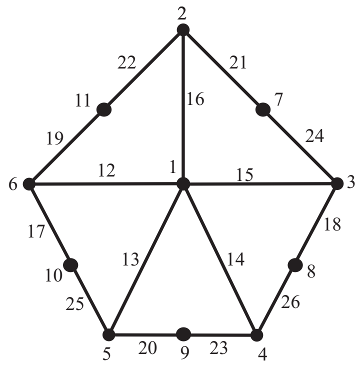

Figure 2: A C_4-supermagic labeling on gear graph or Jahangir graph $J_{5,2}$.

4. Wheel Graph

A wheel graph, denoted by W_n, is a graph obtained by joining n vertices of C_n with one central vertex (Wallis et al. [12]). A wheel graph has $|V(W_n)| = n+1$ and $|E(W_n)| = 2n$, where $n \geq 3$. Lladó and Moragas [4] have proved that a wheel graph W_n for $n \geq 5$ odd, is C_3-supermagic labeling. Here we complete the result of Lladó and Moragas [4] on a wheel graph W_n for n even.

Theorem 4.1. *Any wheel graph W_n for n even and $n \geq 4$ is C_3-supermagic.*

Proof. Let G be the wheel graph W_n. Let $v_1, v_2, \ldots, v_n$ be vertices of C_n, and v_0 be its central vertex of G. Hence $V(G) = \{v_0, v_1, v_2, \ldots, v_n\}$, and $E(G) = \{v_0v_1, v_0v_2, \ldots, v_0v_n, v_1v_2, \ldots, v_{n-1}v_n, v_nv_1\}$. Let $N_i = \{v_i, v_iv_0\}$ for $i \in [1, n]$. Let f be a total labeling of G, and $s(f)$ be the supermagic sum in every subgraph H of G that is isomorphic to C_3. A supermagic sum is obtained by the formula

$$s(f) = f(N_i) + f(N_{i+1}) + f(v_iv_{i+1}) + f(v_0). \tag{1}$$

Furthermore we divide the proof into two cases: $\frac{n}{2}$ odd, and $\frac{n}{2}$ even.

Case 1. $\frac{n}{2}$ is even.

The total labeling f of G is defined as follows.

1. Label each vertex of G other than central vertex by

$$f(v_i) = \begin{cases} \lfloor \frac{i}{2} \rfloor + 1, & \text{for } i \text{ odd; } i \in [1, n] \\ \frac{1}{2}(i + n), & \text{for } i \text{ even; } i \in [1, \frac{n}{2}] \\ \frac{1}{2}(n + i) + 1, & \text{for } i \text{ even; } i \in [\frac{n}{2} + 1, n]. \end{cases}$$

2. Next, label each edge of G, as follows

$$f(v_0v_i) = \begin{cases} 2(n + 1) - i, & \text{for } i \in [1, \frac{n}{2}] \\ 2n - i + 1, & \text{for } i \in [\frac{n}{2} + 1, n - 1] \\ \frac{3}{2}n + 1, & \text{for } i = n \end{cases}$$

and

$$f(v_iv_{i+1}) = \begin{cases} 2(n + 1) + i, & \text{for } i \in [1, \frac{n}{2} - 1] \\ 2n + i + 3, & \text{for } i \in [\frac{n}{2}, n - 2] \\ \frac{5}{2}n + 2, & \text{for } i = n - 1. \end{cases}$$

Then, $f(v_0) = \frac{n}{4} + \lceil \frac{n-1}{2} \rceil + 1$, $f(v_nv_1) = 2n + 2$, $f(N_1) = 2n + 2$, and $f(N_n) = \frac{5}{2}n + 2$. Let $s(f)$ be a constant sum in every subgraph C_3 of G. By Equation (1), we have $s(f) = 6(n + 1) + \frac{3}{4}n + \lceil \frac{n-1}{2} \rceil + 1$. Hence W_n, when $\frac{n}{2}$ is even, is C_3-supermagic labeling.

Case 2. $\frac{n}{2}$ is odd.

The total labeling f of G is defined as follows.

1. Label each vertex as follows

$$f(v_i) = \begin{cases} \lfloor \frac{i}{2} \rfloor + 1, & \text{for } i \text{ odd; } i \in [1, n] \\ \frac{1}{2}(n + i) + 1, & \text{for } i \text{ even; } i \in [1, \frac{n}{2}] \\ \frac{1}{2}(n + i) + 2, & \text{for } i \text{ even; } i \in [\frac{n}{2} + 1, n - 1] \\ \frac{i}{2} + 1, & \text{for } i = n. \end{cases}$$

2. Furthermore, we label each edge of G,

$$f(v_0v_i) = \begin{cases} 2(n+1), & \text{for } i=1 \\ 2(n+1)-i, & \text{for } i \in [2, \frac{n}{2}] \\ 2n-i, & \text{for } i \text{ even}; \ i \in [\frac{n}{2}+1, n-1] \\ 2(n+1)-i, & \text{for } i \text{ odd}; \ i \in [\frac{n}{2}+1, n] \\ \frac{3}{2}n+1, & \text{for } i=n \end{cases}$$

and

$$f(v_iv_{i+1}) = \begin{cases} 2n+1, & \text{for } i=1 \\ 2n+i+1, & \text{for } i \in [2, \frac{n}{2}-1] \\ 2(n+1)+i, & \text{for } i \in [\frac{n}{2}, n-1]. \end{cases}$$

Then, we define $f(v_0) = \frac{3}{2}(\frac{1}{2}n+1)$, $f(v_nv_1) = \frac{5}{2}n+1$, $f(N_1) = 2n+3$, and $f(N_n) = 2n+2$. By Equation (1), we have a constant sum in every subgraph H which is $s(f) = \frac{1}{4}(29n+30)$. Hence, W_n when $\frac{n}{2}$ is odd is C_3-supermagic. This completes the proof of the theorem. □

Figure 3 represents C_3-supermagic labelings on W_4 with $s(f) = 36$ and on W_6 with $s(f) = 51$.

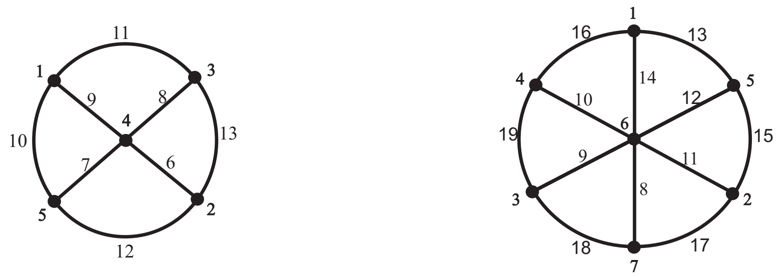

Figure 3: C_3-supermagic labelings on W_4 and W_6

5. Complete Bipartite Graph

According to Wallis [11], the complete bipartite graph on V_1 and V_2 has two disjoint sets of vertices, V_1 and V_2; two vertices are adjacent if and only if they lie in different sets. We write $K_{m,n}$ to mean a complete bipartite graph with m vertices in one side and n at the other. Hence we have $|V(G)| = m+n$ and $|E(G)| = mn$. We proceed to prove that a complete bipartite graph $K_{m,n}$ is C_4-supermagic when $m = 2$ and $n \geq 2$.

Theorem 5.1. *The complete bipartite graph $K_{2,n}$ is C_4-supermagic.*

Proof. Let $G = K_{2,n}$ with $|V(G)| = 2+n$, and $|E(G)| = 2n$. Let H be a subgraph C_4 of G, and c be the number of H-supermagic labelings in G. Let $Z \in [1, 3n+2]$. The elements of Z will be used to label all the vertices and edges of G. Let $s(f)$ be a supermagic sum of any subgraph H in G isomorphic to C_4. We divide the proof into two cases.

Case 1. n is odd.

A total labeling f on $K_{2,n}$ for n odd can be obtained by dividing Z into two sets which are X and Y. Let X be a set of vertices on V_1, where $X = \{x_1, x_2\}$ with $x_1 = 1$ and $x_2 = 2$. Let x, y, z and k be positive integers and a multiset $Y = A \uplus B \uplus C$ with $Y = [x+2, x+k+1] \uplus [y+1, y+k] \uplus [z+1, z+k]$, $Y \in [3, 3n+2]$. Let Y be a set of $Y_i = \{a_i, b_i, c_i\}$, $i \in [1, k]$ as in Lemma 2.4. By applying Lemma 2.4, we set $x = 1, y = 2+n$ and $z = 2+2n$, Y is k-balanced, with labeling of vertices of V_2 by elements a_i of Y_i, elements b_i of Y_i to label edges incident to x_1, and c_i of Y_i to label edges incident to x_2. Hence, for Y, which is Y_i with $i \in [1, k]$, we have $\sum Y_i = 4n + 7 + \lceil \frac{n}{2} \rceil$. If H is isomorphic to C_4 then for every $i \in [1, k]$, $f(H_i) = \sum X + 2 \sum Y_i$ is constant since Y_i is a balanced subset of Y. Use the smaller labels in every a_i of Y_i. It is clear that a supermagic sum is

$$f(H) = 8n + 2\lceil \frac{n}{2} \rceil + 17. \tag{2}$$

Case 2. n is even.

A total labeling f on $K_{2,n}$ for n even is obtained by dividing Z into two sets X and Y, and let X be a set of vertices of V_1, where $X = \{x_1, x_2\}$ with $x_1 = 1$ and $x_2 = \frac{n}{2} + 2$. Let x, y, z and k be positive integers and multiset $Y = A \uplus B \uplus C$ with $Y = [x+1, x+k+1] \setminus \{k\} \uplus [y+1, y+k] \uplus [z+1, z+k]$, $Y \in [3, 3n+2]$. Let Y be a set of $Y_i = \{a_i, b_i, c_i\}$, $i \in [1, k]$ as in Lemma 2.5 with $x = 1, y = 2+n, z = 2+2n$ and $k = n$. By using Lemma 2.5, we label vertices of V_2 by elements a_i of Y_i, elements b_i to label edges incident to x_1 of Y_i, and elements c_i to label edges incident to x_2. It is obvious that Y is k-balanced. Hence we have $\sum Y_i = 4n + 7 + \frac{n}{2}$. If H is isomorphic to C_4 then for every $i \in [1, k]$, $f(H_i) = \sum X + 2 \sum Y_i$ is constant because Y_i is a balanced subset of Y. Use the smaller labels in every a_i of Y_i. It is obvious that a supermagic sum is

$$f(H) = 9n + \frac{n}{2} + 17. \tag{3}$$

This completes the proof of the theorem. □

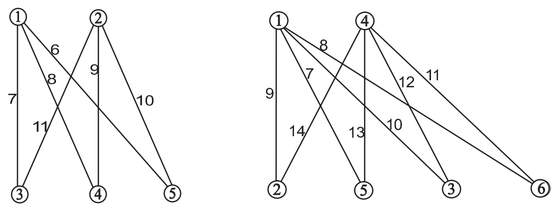

Figure 4: A C_4-supermagic labeling on complete bipartite graph $K_{2,3}$ (left) and $K_{2,4}$ (right)

The following problem still remain open.

Open Problem. Does the complete bipartite graph $K_{m,n}, m \leq n, m > 2$ admit a C_{2m}-supermagic labeling?

References

[1] J. A. Gallian, A dynamic survey of graph labeling, *Electron. J. Combin.*, **18** (2011), # DS6.

[2] A. Gutiérrez, A. Lladó, Magic Coverings, *J. Combin. Math. Combin. Comput.*, **55** (2005), 43–56.

[3] P. Jeyanthi and P. Selvagopal, More classes of H-supermagic Graphs, *Intern. J. of Algorithms, Computing and Mathematics* **3**(1) (2010), 93–108.

[4] A. Lladó and J. Moragas, Cycle-magic Graphs, *Discrete Math.*, **307** (23) (2007), 2925–2933

[5] T. K. Maryati, E. T. Baskoro, and A. N., M. Salman, P_h-supermagic labelings of some trees, *J. Combin. Math. Combin. Comput.*, **65** (2008), 197–204.

[6] T. K. Maryati, A. N. M. Salman, E. T. Baskoro, J. Ryan and M. Miller, On H-supermagic labelings for certain schakles and amalgamations of a connected graph, *Util. Math.*, **83** (2010), 333–342.

[7] T. K. Maryati, Karakterisasi graf H ajaib dan graf H ajaib super, *Disertasi S3*, Insitut Teknologi Bandung, 2011.

[8] A. A. G. Ngurah, A. N. M. Salman, and L. Susilowati, H-supermagic labeling of graphs, *Discrete Math.*, **310**(8) (2010), 1293–1300.

[9] M. Roswitha and E. T. Baskoro, H-magic covering of some classes of graphs, *AIP Conf. Proc. on ICREM5*, ITB Bandung, **1450** (2012), 135–138.

[10] A. N. M. Salman and T. K. Maryati, On graph-(super)magic labelings of a path-amalgamation of isomorphic graphs, *Proc. of the 6thIMT-GT Conference on Mathematics, Statistics and its Applications ICMSA* 2010, Universiti Tunku Abdul Rahman, Kuala Lumpur, Malaysia, 2010.

[11] W. D. Wallis, *Magic Graph*, Birkhäuser, Boston, Basel, Berlin, 2001.

[12] W. D. Wallis, E. T. Baskoro, M. Miller and Slamin, Edge magic total labeling, *Australas J. Combin*, **22** (2000), 177–190.