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Mean labeling of edge linked cyclic snakes

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Abstract

A vertex labeling of a graph G is an assignment f of labels to the vertices of G that induces a label for each edge uv depending on the vertex labels. A *mean labeling* f is an injection from V to the set $\{0, 1, 2, \ldots, q\}$ that induces for each edge uv the label $\left\lceil \frac{f(u)+f(v)}{2} \right\rceil$ such that the set of edge labels is $\{1, 2, \ldots, q\}$. In this paper we present mean labeling of edge linked cyclic snakes and generalised edge linked cyclic snakes.

Keywords: mean labeling, cycles, cyclic snakes, edge linked cyclic snakes.

2010 Mathematics Subject Classification: 05C78.

1. Introduction

A vertex labeling of a graph G is an assignment f of labels to the vertices of G that induces a label for each edge uv depending on the vertex labels. Let $G = (V, E)$ be a simple graph with p vertices and q edges. A *mean labeling* f is an injection from V to the set $\{0, 1, 2, \ldots, q\}$ that induces for each edge uv the label $\left\lceil \frac{f(u)+f(v)}{2} \right\rceil$ such that the set of edge labels is $\{1, 2, ..., q\}$. Mean labeling was introduced by Somasundaram et al. [7]. Further results on mean labeling are given in [3], [4], [5], [6], [8], [9], [10], and [11]. A graph that has a mean labeling is known as *mean graph.* For a summary on various graph labelings see the dynamic survey of graph labeling by Gallian [2].

A kC_n-snake has been defined as a connected graph in which all the blocks are isomorphic to the cycle C_n and the block-cut point graph is a path. Let P be the path of minimum length that contains all the cut vertices of a kC_n-snake. Barrientos [1] has proved that any kC_n-snake is represented by a string $s_1, s_2, \ldots, s_{k-2}$ of integers of length $k-2$ where the i^{th} integer, s_i, on the string is the distance between i^{th} and $(i+1)^{th}$ cut vertices on the path P from one extreme and is taken from $S_n = \{1, 2, \dots, \lfloor \frac{n}{2} \rfloor\}$. This representation is not unique, because it depends on the extreme of P taken. This problem can be avoided by considering the strings obtained for both extremes as the same. Lourdusamy et al. [3] have proved that kC_n-snakes are mean graphs for all $k \geq 1$ and $n \geq 3$. In this paper we define edge linked cyclic snake, as an edge analogue of kC_n-snake, and generalised edge linked cyclic snakes and analyse the conditions under which they are mean graphs.

2. Main Results

Consider the following edge analogue of kC_n-snake.

Definition 2.1. *A connected graph G obtained from k copies of the cycle C_n, where $n \geq 4$, by identifying an edge of the $(i+1)^{th}$ copy, called link-i , to an edge of the i^{th} copy for each i, $1 \leq i \leq k-1$, in such a way that consecutive links are not adjacent is called an edge linked cyclic snake-$EL(kC_n)$-snake.*

For example an $EL(7C_6)$-snake is shown in Figure 1.

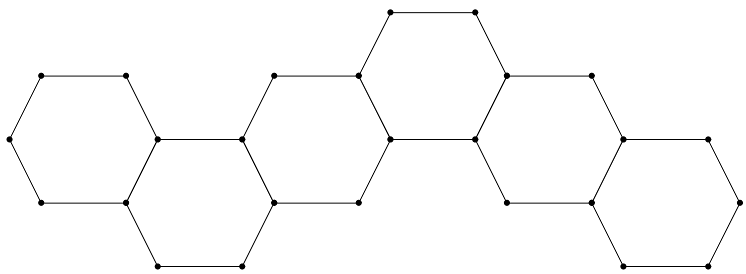

Figure 1: *An $EL(7C_6)$-snake*

It is interesting to note that there is only one $EL(kC_4)$-snake, the ladder $P_n \times P_2$ which is a mean graph [4]. For completeness, we present the mean labeling of $EL(kC_4)$-snake. Let $G = EL(kC_4)$-snake. First we label the i^{th} copy of C_4 for each i using the labeling of C_4 shown in Figure 2 with $x = 3(i-1)$. Second the edge labels induced on the i^{th} copy of C_4 are $3(i-1)+1, 3(i-1)+2, 3(i-1)+3, 3(i-1)+4$. Finally i^{th} and $(i+1)^{th}$ copies are connected by their edges with common label $3i+1$ to obtain a mean labeling of G. For example a mean labeling of an $EL(7C_4)$-snake is shown in Figure 3.

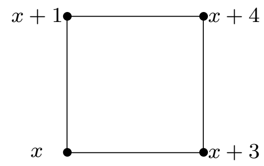

Figure 2: A labeling of C_4

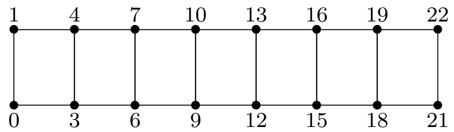

Figure 3: A mean labeling of an $EL(7C_4)$-snake

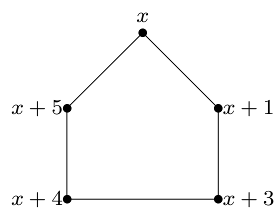

Figure 4: A labeling of C_5

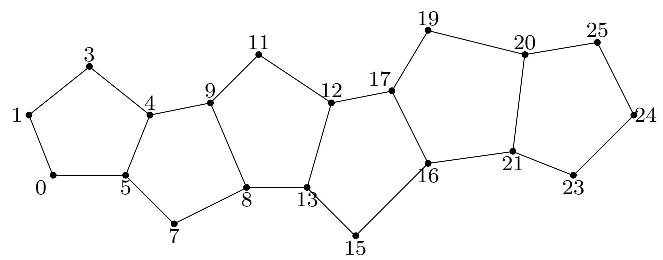

Figure 5: A mean labeling of $EL(6C_5)$-snake

Consider k copies of the cycle C_5. For each i, first we label the i^{th} copy of C_5 using the labeling of C_5 shown in Figure 4 with $x = 4(i-1)$. Then the edge labels induced on the i^{th} copy of C_5 are $4(i-1)+1, 4(i-1)+2, 4(i-1)+3, 4(i-1)+4, 4(i-1)+5$. Finally i^{th} and $(i+1)^{th}$ copies are connected by their edges with common label $4i+1$ to obtain a mean labeling of an $EL(kC_5)$-snake. A mean labeling of an $EL(6C_5)$-snake thus obtained is shown in Figure 5. So far, we have given mean labeling of the $EL(kC_4)$-snake and a $EL(kC_5)$-snake. Next, we proceed to present mean labeling of $EL(kC_n)$-snakes for even and odd values of n separately.

Vaidya et al. [11] have proved that $EL(2C_n)$-snake, two copies of C_n sharing a common edge, admits mean labeling. The way to construct an $EL(2C_n)$-snake is unique. For $k \geq 3$, a copy of C_n can be attached in $n-3$ ways to an $EL((k-1)C_n)$-snake to obtain an $EL(kC_n)$-snake. Let G be an $EL(kC_n)$-snake. Consider a path P of minimal length that contains all the links of G. Obviously both ends of P are links. Beginning from one of its extreme links, it is possible to construct a string $s_1^{'}, s_2^{'}, s_3^{'}, ..., s_{k-2}^{'}$ of $k-2$ integers where the i^{th} integer, $s_i^{'}$, on the string is the number of edges that separates the *link-i* from the *link-(i+1)* of G on the path P. For each i, $1 \leq i \leq k-1$, let, u_iv_i denote the *link-i* of G on the path P, so that the integer $s_i^{'}$ becomes the length of the $v_i - u_{i+1}$ path on P. As there are $n-3$ different ways to connect the $(i+1)^{th}$ copy of C_n to the i^{th} copy, $s_i^{'}$ is taken from $S_n^{'} = \{1, 2, 3, \ldots, n-3\}$. Until now, this representation is not unique, because it depends on the extreme of P taken and there are exactly two such paths as P. But, the four strings obtained for both ends of each of the two paths are the same, in the sense that one is obtained from the other by means of one of the following operations:

1. reversing the string,
2. replacing each s_i' on the string by $n-2-s_i^{'}$,
3. replacing each s_i' on the string by $n-2-s_i^{'}$ and reversing it.

Thus without loss of generality we assume that any $EL(kC_n)$-snake is uniquely represented by a string. This is illustrated in the following example.

Example 2.2. *Consider the $G = EL(5C_7)$-snake shown in Figure 6. Consider the path P_1 : abcdefghijklmn of minimal length that contains all the links of G. It is easy to observe that ab, de, hi, and mn are the links of G. Beginning from the link ab, we observe that two edges separate link-1(ab) from link-2(de), thus we have $s_1^{'} = 2$. In a similar way the integers $s_2^{'} = 3$, $s_3^{'} = 4$ can be obtained. Hence $2, 3, 4$ is a string attached to G. Had we constructed the string by beginning from the link nm, the string would have been $4, 3, 2$. Similarly $3, 2, 1$ and $1, 2, 3$ would have been the strings had we constructed the string by beginning from the links ba and nm respectively using the path P_2 : barsedtihnm. It is easy to observe that one string can be obtained from the other by means of any one of the operations mentioned earlier, in the construction of the string. These four strings are considered to be the same and we use one of them to represent G.*

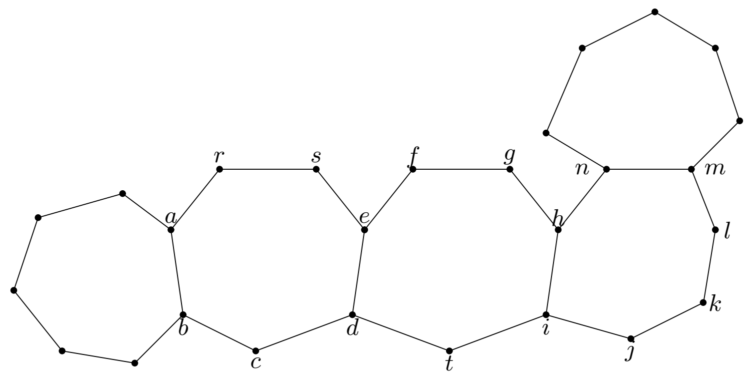

Figure 6: *An $EL(5C_7)$-snake*

Notation 2.3. *Let $G = EL(kC_n)$-snake with the string $s_1^{'}, s_2^{'}, s_3^{'}, ..., s_{k-2}^{'}$. The i^{th} integer $s_i^{'}$ is said to be low if $s_i^{'} \leq \lceil \frac{n}{2} \rceil - 2$ and high if $s_i^{'} \geq \lfloor \frac{n}{2} \rfloor$. For each $i, 1 \leq i \leq k-2$, let d_i denote the integer which is the minimum of $s_i^{'}$ and $n-2-s_i^{'}$. For each i, $1 \leq i \leq k-3$, we define t_i as follows:*

Case 1. $n \cong 0(mod\ 2)$.

For each i where $d_i \neq \lfloor \frac{n}{2} \rfloor - 1$, let $j \geq i$ be the least positive integer such that $d_j \neq \lfloor \frac{n}{2} \rfloor - 1$. Consider the following cases:

1. *both $s_i^{'}$ and $s_j^{'}$ are low and $i \ncong j(mod\ 2)$*
2. *both $s_i^{'}$ and $s_j^{'}$ are high and $i \ncong j(mod\ 2)$*
3. *$s_i^{'}$ is low and $s_j^{'}$ is high and $i \equiv j(mod\ 2)$*
4. *$s_i^{'}$ is high and $s_j^{'}$ is low and $i \equiv j(mod\ 2)$*

The value of t_i is defined to be 2 if one of the four conditions (1), (2), (3), and (4) holds and 1 otherwise. From the definition of t_i it is obvious that $(t_i, d_i) \neq (2, \lfloor \frac{n}{2} \rfloor - 1)$ for any i.

Case 2. $n \cong 1(mod\ 2)$.

The value of t_i is defined to be 2, if both $s_i^{'}$ and $s_{i+1}^{'}$ are low or both $s_i^{'}$ and $s_{i+1}^{'}$ are high, and 1 otherwise.

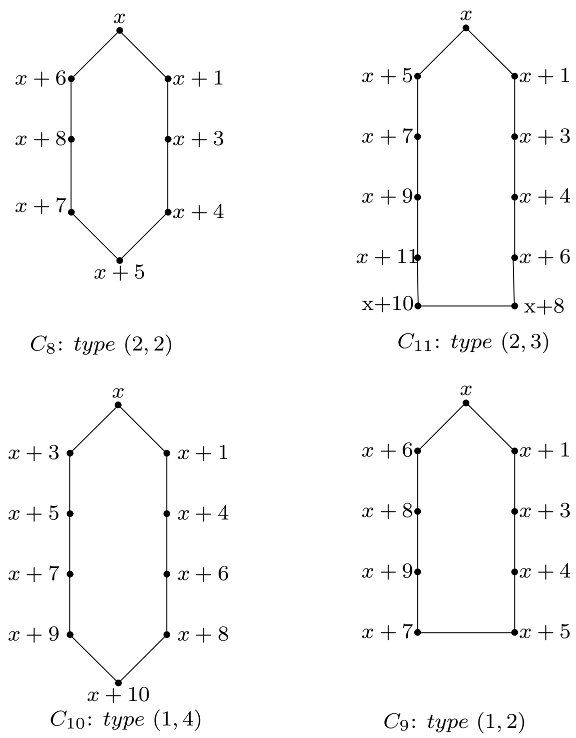

Figure 7: *Some type (t, d) labelings of C_n*

Notation 2.4. *Let $v_0, v_1, v_2, ..., v_{n-1}$ be the consecutive vertices of the cycle C_n where $n \geq 4$. For $t \in \{1, 2\}$, an injective function from $V(C_n)$ to the set $\{x, x+1, x+2, ..., x+n\}$ that places the labels $x+1, x, x+n+t-2$, and $x+n+1-t$ respectively on the vertices v_{i-1}, v_i, v_{i+d}, and v_{i+d+1}, the suffices are taken modulo n and the vertices v_i and v_{i+d} are at a distance d apart, such that the set of induced edge labels is $\{x+1, x+2, ..., x+n\}$ is called a type (t,d) labeling of the cycle C_n. To illustrate this, some type (t,d) labelings of the cycle C_n for various values of n, t, and d are shown in Figure 7.*

Let $1 \leq d \leq n-1$ and $\zeta = n-d$. Consider the labeling $g_{t,d} : V(C_{2n}) \longrightarrow \{x, x+1, ..., x+2n\}$ defined by the following three steps:

Step 1 Assign the numbers $x+1, x, x+2n+t-2$, and $x+2n+1-t$ respectively on the vertices v_{2n-1}, v_0, v_d, and v_{d+1}.

Step 2 For $1 \leq i \leq d-1$, the vertices v_{d-i} and v_{d+i+1} are assigned respectively the numbers $x+2n+t-2-2i$ and $x+2n+1-t-2i$.

Step 3 The remaining unlabeled vertices are assigned numbers as follows: The numbers left unassigned to any vertex in step 1 and step 2 except $x+\zeta$ are arranged as an increasing sequence $\alpha_1, \alpha_2, , ...\alpha_{2n-2d-2}$ and α_k is assigned to v_{2n-k-1}.

Clearly, except for the case where $t = 2$ and $d = n-1$, the vertices of C_{2n} receive distinct labels and the edge labels induced are $x+1, x+2, \ldots, x+2n$. If $x = 0$, the labeling is a mean labeling of C_{2n}. Thus $g_{t,d}$ is a *type (t,d)* labeling of C_{2n} except for $(t,d) = (2, n-1)$. To illustrate this, *type (t,d)* labelings of C_6 except for $(t,d) = (2,2)$ are shown in Figure 8.

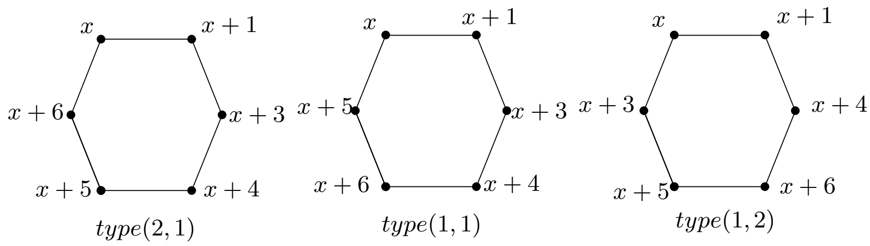

Figure 8: *type (t, d) labelings of* C_6

Thus we have proved the following two lemma:

Lemma 2.5. *If* $1 \leq d \leq n-1$ *and* $t \in \{1, 2\}$*, the cycle* C_{2n} *has type (t,d) labeling for every ordered pair* (t,d) *except* $(2, n-1)$*.*

Theorem 2.6. *The* $EL(kC_{2n})$*-snake is a mean graph for all* $k \geq 1$*.*

Proof. Let $G = EL(kC_{2n})$-snake and $s_1', s_2', s_3', ..., s_{k-2}'$ be the string attached to it. Let $C_{2n,0}, C_{2n,1}, ..., C_{2n,k-1}$ be the consecutive copies of the cycle $C_{2n}, n \geq 2$ in G. First we label the vertices of each $C_{2n,i}$, $0 \leq i \leq k-1$, by a *type (t,d)* labeling as given in Table 1. This labeling scheme is possible because $(t_i, d_i) \neq (2, n-1)$ for any i.

Copy of C_{2n-1}	Type used	Value of x
$C_{2n,0}$	any type	0
$C_{2n,i}, 1 \leq i \leq k-3$	*type*(t_i, d_i)	$(2n-1)i$
$C_{2n,k-2}$	*type*$(\ 1, d_i)$	$((2n-1)(k-2)$
$C_{2n,k-1}$	any type	$(2n-1)(k-1)$

Table 1:

We observe that the edge labels induced on $C_{2n,i}$ are $(2n-1)i+1, (2n-1)i+2, ..., (2n-1)i+2n$ and $C_{2n,i-1}$ and $C_{2n,i}$ have the common edge label $(2n-1)i+1$ which is induced by the vertex labels $(2n-1)i$ and $(2n-1)i+1$ on both copies. Then the $EL(kC_{2n})$-snake obtained by connecting $C_{2n,i}$ and $C_{2n,i-1}$ by merging their edges having the common label $(2n-1)i+1$ is a mean graph. But, the values of d_i and t_i are so defined that the string attached to this $EL(kC_{2n})$-snake is $s_1^{'}, s_2^{'}, s_3^{'}, \ldots, s_{k-2}^{'}$. Hence the theorem. □

The following example illustrates the methods followed in Theorem 2.6.

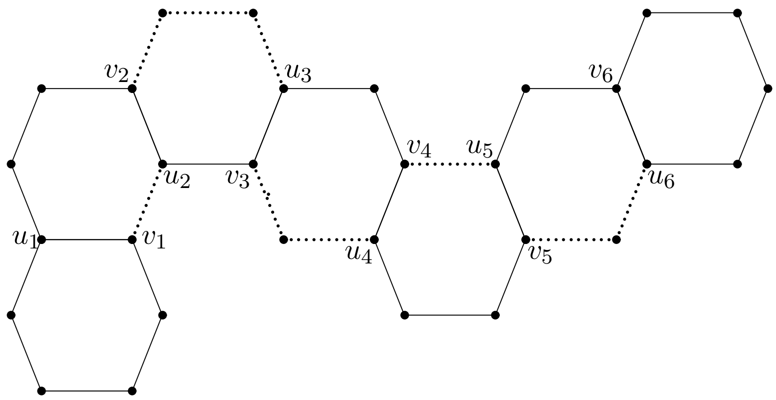

Figure 9:

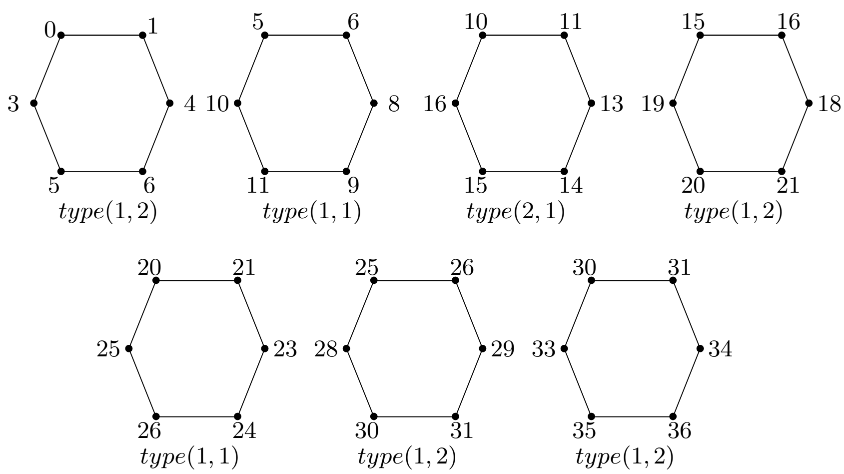

Figure 10: *type(t, d)) labeling of consecutive copies of* C_6

Example 2.7. *Consider the $G = EL(7C_6)$-snake, shown in Figure 9. We take the $u_1 - v_6$ path of minimal length that contains all the links and traversed through dotted edges to get its string. The string attached to this graph is 1, 3, 2, 1, 2. That is $s_1^{'} = 1$, $s_2^{'} = 3$, $s_3^{'} = 2$, $s_4^{'} = 1$, and $s_5^{'} = 2$. First we find the values of d_i for $1 \leq i \leq 5$. The value of d_1 is the minimum of $s_1^{'} = 1$ and $2n - 2 - s_1^{'} = 3$,that is $d_1 = 1$. Similarly $d_2 = 1$, $d_3 = 2$, $d_4 = 1$, and $d_5 = 2$ are obtained. Second, we find the values of t_i for $1 \leq i \leq 4$. When $i = 1$, $d_1 \neq n - 1$ and $j = 2$ is the least positive integer such that $d_2 \neq n - 1$. We observe that non of the four conditions stated in the definition of t_i hold here, so, $t_1 = 1$. Similarly, $t_2 = 2$, $t_3 = 1$, $t_4 = 1$ are computed. Third, the consecutive copies of C_6 are labeled according to Table 1.*

Copy of C_{2n}	Type used	Value of x
1	any type	0
2	$type(1,1)$	5
3	$type(2\ ,1)$	10
4	$type(1,2)$	15
5	$type(1,1)$	20
6	$type(1,2)$	25
7	any type	30

Table 2:

Finally $C_{6,1}$ and $C_{6,0}$ are connected by their edges with common label 6, $C_{6,2}$ and $C_{6,1}$ are connected by their edges with common label 11, $C_{6,3}$ and $C_{6,2}$ are connected by their edges with common label 16, $C_{6,4}$ and $C_{6,3}$ are connected by their edges with common label 21, $C_{6,5}$ and $C_{6,4}$ are connected by their edges with common label 26, and $C_{6,6}$ and $C_{6,5}$ are connected by their edges with common label 31 to obtain a mean graph shown in Figure 11. It is interesting to observe that this graph is an $EL(7C_6)$-snake with the string 1, 3, 2, 1, 2, same as that of the $EL(7C_6)$-snake shown in Figure 9. Thus Figure 11 gives a mean labeling of the $EL(7C_6)$-snake shown in Figure 9.

Let $1 \leq d \leq n - 2$ and $\zeta = n - d + t - 2$. Consider the labeling $g_{t,d} : V(C_{2n-1}) \longrightarrow \{x, x + 1, \ldots, x + 2n - 1\}$ defined by the following three steps:

Step 1 Assign the numbers $x + 1, x, x + 2n + t - 3$, and $x + 2n - t$ respectively on the vertices v_{2n-2}, v_0, v_d, and v_{d+1}.

Step 2 For $1 \leq i \leq d - 1$, the vertices v_{d-i} and v_{d+i+1} are assigned respectively the numbers $x + 2n + t - 3 - 2i$ and $x + 2n - t - 2i$.

Step 3 The remaining unlabeled vertices are assigned numbers as follows: The numbers left unassigned to any vertex in step 1 and step 2 except $x + \zeta$ are arranged as an increasing sequence $\alpha_1, \alpha_2, , ...\alpha_{2n-2d-3}$ and α_k is assigned to v_{2n-k-2}.

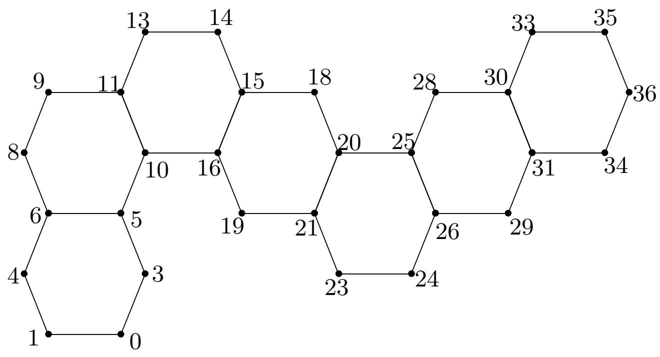

Figure 11: *A mean labeling of* $EL(7C_6)$*-snake*

Clearly, except for $t = 1$ and $d = n - 2$, the vertices of C_{2n-1} receive distinct labels and the edge labels induced are $x + 1, x + 2, \ldots, x + 2n - 1$. If $x = 0$, the labeling is a mean labeling of C_{2n-1}. Observe that $g_{t,d}$ is a *type (t,d)* labeling of C_{2n-1} except for $(t, d) = (1, n - 2)$. To illustrate this, *type (t,d)* labelings of C_7 except for $(t, d) = (1, 2)$ are shown in Figure 12. Thus we proved the following lemma.

Lemma 2.8. *If* $1 \leq d \leq n - 2$ *and* $t \in \{1, 2\}$*, the cycle* C_{2n-1} *has type* (t, d) *labeling for every ordered pair* (t, d) *except* $(1, n - 2)$*.*

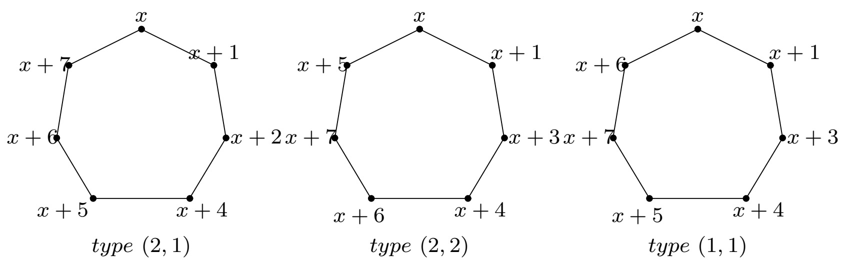

Figure 12: *type (t, d) labelings of* C_7

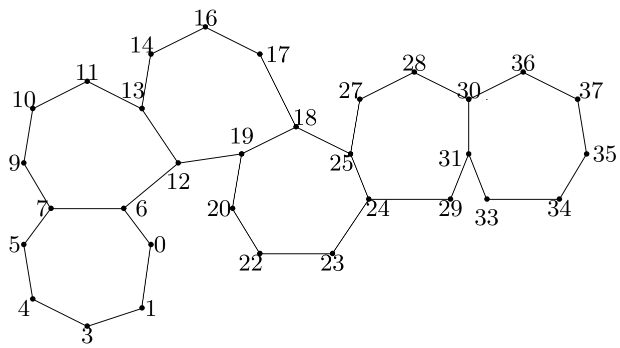

Figure 13: *A mean labeling of* $EL(6C_7)$*-snake*

Theorem 2.9. *The* $EL(kC_{2n-1})$*-snake is a mean graph for all* $k \geq 1$ *if* $(t_i, d_i) \neq (1, n-2)$ *for every* i, $1 \leq i \leq k-3$.

Proof. Let G be any $EL(kC_{2n-1})$-snake with the string $s_1^{'}, s_2^{'}, s_3^{'}, ..., s_{k-2}^{'}$. Let $C_{2n-1,0}, C_{2n-1,1}, C_{2n-1,2}, ..., C_{2n-1,k-1}$ be the consecutive copies of C_{2n-1} in G. First we label the consecutive copies of C_{2n-1} using *type (t,d)* labeling as given in Table 3.

Copy of C_{2n-1}	Type used	Value of x
$C_{2n-1,0}$	any type	0
$C_{2n-1,i}, 1 \leq i \leq k-3$	$type(t_i, d_i)$	$(2n-2)i$
$C_{2n-1,k-2}$	$type(\ 2, d_i)$	$(2n-2)(k-2)$
$C_{2n-1,k-1}$	any type	$(2n-2)(k-1)$

Table 3:

This labeling scheme is possible because $(t_i, d_i) \neq (1, n-2)$ for any i. Second the edge labels induced on $C_{2n-1,i}$ are $(2n-2)i+1, (2n-2)i+2, \ldots, (2n-2)i+2n-1$. Finally $C_{2n-1,i}$ and $C_{2n-1,i-1}$ are connected by merging their edges having the common label $(2n-2)i$ +1 to obtain a mean labeling of G as explained in the proof of Theorem 2.6. □

For example a mean labeling of an $EL(6C_7)$-snake is shown in Figure 13.

Definition 2.10. *A connected graph* G *obtained from* k *disjoint cycles* C_{n_0}, C_{n_1}, ..., $C_{n_{k-1}}$*, where* $n_i \geq 4$ *for each* i*, by identifying an edge of* C_{n_i}*, called link-i, to an edge of* $C_{n_{i-1}}$ *for* $1 \leq i \leq k-1$ *in such a way that consecutive links are not adjacent is called a generalised edge linked cyclic snake-*$ELCS(n_1, n_2, n_3, ..., n_k)$.

By applying the same methods used to obtain the string of an$EL(kC_n)$-snake, we can show that any generalised edge linked cyclic snake can be represented by a string of integers, $s_1^{'}, s_2^{'}, s_3^{'}, ..., s_{k-2}^{'}$ of length $k-2$, where the i^{th} integer, $s_i^{'}$, on the string is the number of edges that separates the i^{th} and $(i+1)^{th}$ links of G on the path P and $s_i^{'} \in \mathcal{S}_{n_i}^{'}$ for $1 \leq i \leq k-2$. An integer in the string, $s_i^{'}$, is said to be *low* if $s_i^{'} \leq \lceil \frac{n_i}{2} \rceil - 2$, and *high* if $s_i^{'} \geq \lfloor \frac{n_i}{2} \rfloor$. Let d_i denote the integer which is the minimum of $s_i^{'}$ and $n_i - 2 - s_i^{'}$. For each i, $1 \leq i \leq k-3$, we define t_i as follows: For each i, where n_i is even and $d_i \neq \lfloor \frac{n_i}{2} \rfloor - 1$, let $j \geq i$ be the least positive integer such that n_j is odd or $n_j$is even and $d_j \neq \lfloor \frac{n_j}{2} \rfloor - 1$, and for each i, where n_i is odd, let $j \geq i$ be the least positive integer such that n_j is odd or n_j is even and $d_j \neq \lfloor \frac{n_j}{2} \rfloor - 1$. Consider the following cases:

1. both $s_i^{'}$ and $s_j^{'}$ are low and $i \not\equiv j (mod\ 2)$
2. both $s_i^{'}$ and $s_j^{'}$ are high and $i \not\equiv j (mod\ 2)$
3. $s_i^{'}$ is low and $s_j^{'}$ is high and $i \equiv\ j (mod\ 2)$
4. $s_i^{'}$ is high and $s_j^{'}$ is low and $i \equiv\ j (mod\ 2)$

The value of t_i is defined to be 2 if one of the four conditions (1), (2), (3), and (4) holds and 1 otherwise.

Theorem 2.11. *The $ELCS(n_1, n_2, n_3, ..., n_k)$ is a mean graph for all $k \geq 1$ if $(t_i, d_i) \neq (1, \lfloor \frac{n_i}{2} \rfloor - 1)$ whenever $n_i \equiv\ 1(\ mod\ 2)$.*

Proof. Let $G = ELCS(n_1, n_2, n_3, ..., n_k)$. Let C_{n_0}, C_{n_1}, ..., $C_{n_{k-1}}$ be the consecutive cycles of G where $n_i \geq 4$ for each i and $s_1^{'}, s_2^{'}, s_3^{'}, ..., s_{k-2}^{'}$ be the string attached to it. First we label the vertices of each C_{n_i}, $0 \leq i \leq k-1$, by a *type (t,d)* labeling as given in Table 4. This labeling scheme is possible because the required labelings are available in Lemma 2.5 and Lemma 2.8.

C_{n_i}	Type used	Value of x
C_{n_0}	any type	0
$C_{n_i}, 1 \leq i \leq k-3$	*type*(t_i, d_i)	$\sum_0^{i-1}(n_r - 1)$
$C_{n_{k-2}}$	*type*$(1, d_i)$ if n_{k-2} is even *type*$(2, d_i)$ if n_{k-2} is odd	$\sum_0^{k-3}(n_r - 1)$
$C_{n_{k-1}}$	any type	$\sum_0^{k-2}(n_r - 1)$

Table 4:

We observe that the edge labels induced on C_{n_0} are $1, 2, 3, ...n_0$ and the edge labels induced on C_{n_i}, for $i \geq 1$ are $\sum_0^{i-1}(n_r - 1) + 1, \sum_0^{i-1}(n_r - 1) + 2, ..., \sum_0^{i-1}(n_r - 1) + n_i$.

Also $C_{n_{i-1}}$ and C_{n_i} have the common edge label $\sum_0^i (n_r - 1) + 1$ which is induced by the vertex labels $\sum_0^i (n_r - 1)$ and $\sum_0^i (n_r - 1) + 1$ on both copies. Finally, C_{n_i} and $C_{n_{i-1}}$ are connected by merging their edges having the common label to obtain a mean labeling of G as explained in Theorem 2.6 and Theorem 2.9. $\square$

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