AKCE J. Graphs. Combin., **1,** No. 1 (2004) 41-49

On the Self-Dual Codes over *GF* (7) Generated by Hadamard Matrices of Order 28

Takehisa Wakabayashi

Graduate School of Science and Engineering, Yamagata University, Yamagata 990-8560, Japan
email: gs03111@kdw.kj.yamagata-u.ac.jp

Communicated by: Mariko Hagita
Received: June 03, 2004

Abstract

It is known that there are exactly 487 inequivalent Hadamard matrices of order 28. In this note, we show that the 487 self-dual codes over *GF* (7) generated by the 487 inequivalent Hadamard matrices of order 28 are inequivalent.

Keywords: Code over *GF* (7), Self-dual code and Hadamard matrix

AMS Subject Classification: 94B05, 94B25, 05B20

1 Introduction

An $n \times n$ matrix H with entries from $\{1,-1\}$ satisfying $HH^T = nI_n$ is called a Hadamard matrix of order *n*, where H^T is the transposed matrix of H and I_n is the identity matrix of order *n*. An $n \times n$ matrix with exactly one entry from $\{1,-1\}$ in each row and column and all other entries 0 is called a signed permutation matrix. Two Hadamard matrices H and H' are called equivalent if there are signed permutation matrices Q and R such that $H' = QHR$. Hadamard matrices of order $n \le 28$ are completely classified (cf. [1, Section 7.11]). For example, there are exactly 487 inequivalent Hadamard matrices of order 28 [4].

Let *p* be a prime number, $GF(p)$ be the finite field with *p* elements. A *k*-dimensional subspace C of $V = GF(p)^n$ is called an $[n,k]$ code over $GF(p)$. A matrix *G* whose rows span *C* is called a generator matrix of *C*. *G* is in standard form if $G = (I_k \mid A)$ using some matrix *A*. For $x = (x_1, x_2,..., x_n) \in V$, $wt(x) = \#\{i \mid x_i \ne 0\}$ is called the weight of *x*. The minimum weight of *C* is $\min\{wt(x) \mid x \in C, x \ne 0\}$. If *C* has minimum weight *d*, then *C* is called an $[n,k,d]$ code. Let $A_i\,(i = 0,1,...,n)$ denote the number of codewords of weight *i*, then $A_0, A_1,..., A_n$ are called the weight distribution of *C*. For $x = (x_1, x_2,..., x_n)$, $y = (y_1, y_2,..., y_n) \in V$, $x \cdot y = \sum_{i=1}^{n} x_i y_i$ denotes their inner product. The dual code of *C* is $C^\perp = \{y \in V \mid x \cdot y = 0 (\forall x \in C)\}$. *C* is called a *self-dual code* if $C = C^\perp$. Two codes *C* and C' are equivalent if there is a signed permutation matrix *P* $\left(\text{over}\, GF(p)\right)$ with $C' = CP = \{xP \mid x \in C\}$ [7].

Let H be a Hadamard matrix of order n and p be a prime number. If p divides n and p^2 does not divide n, then the code that generated by rows of H is a self-dual code over $GF(p)$ [7] (see also [1, p. 287, Remark (6)]). It is known that there are exactly 60 inequivalent Hadamard matrices of order 24, and these matrices generated 9 inequivalent self-dual codes over $GF(3)$ [5]. In this note, we consider the next case that $n = 28$ and $p = 7$.

2 Results

We label the 487 Hadamard matrices of order 28 in [8] as $H_1, H_2, ..., H_{487}$. Let $C(H_i)$ denote the self-dual code over $GF(7)$ generated by H_i. We calculate the minimum weights of the 487 codes by MAGMA. From this calculation, only the code $C(H_{478})$ has minimum weight 10 and the other codes have minimum weight 9.

At first, we determine the weight distributions of the 487 codes. By the Pless power moment [6, Section 8.3], if the numbers $A_0, A_1, ..., A_{14}$ of $C(H_i)$ are known then the weight distribution of $C(H_i)$ is determined. Let $x = (y, z) \in C(H_i)$, where y consists of the first 14 coordinates of x and z consists of the last 14 coordinates of x. Suppose that a generator matrix of $C(H_i)$ is in standard form $(I_{14} \mid A)$. When $wt(x) = 2i$, if $wt(y) = \alpha \le i$ then x can be expressed as a linear combination of α rows of $(I_{14} \mid A)$, and if $wt(y) = \beta \ge i+1$ then x can be expressed as a linear combination of $2i - \beta$ rows of $(A^T \mid -I_{14})$. When $wt(x) = 2i-1$, if $wt(y) = \alpha \le i-1$ then x can be expressed as a linear combination of α rows of $(I_{14} \mid A)$, and if $wt(y) = \beta \ge i$ then x can be expressed as a linear combination of $2i - \beta - 1$ rows of $(A^T \mid -I_{14})$ (cf. [3]). Using this fact, we calculate the numbers $A_0, A_1, ..., A_{14}$ by a program written in *C*. From this calculation, we obtain the weight distributions of the 487 codes and the numbers $A_9, A_{10}, ..., A_{14}$ are listed in Appendix. There are 191 pairs of two self-dual codes $C(H_i)$ and $C(H_j)$ with the same weight distribution, $C(H_{311})$, $C(H_{375})$, $C(H_{422})$ and $C(H_{429})$ have the same weight distribution, and the other self-dual codes have distinct weight distributions.

Let $M_t = (m_{ij})$ be the $A_t \times n$ matrix with rows composed of codewords of weight t in C, where C is a self-dual code of length n. For an integer k $(1 \le k \le n)$, let $n_t(j_1, j_2, ..., j_k)$ be the number of r $(1 \le r \le A_t)$ such that $m_{rj_1}, m_{rj_2}, ..., m_{rj_k} \ne 0$ over $\mathbf{Z}$ for $1 \le j_1 < ... < j_k \le n$. Let $S_t(k)$ be the increasing sequence of the numbers $nt(j_1, j_2, ..., j_k)$ for any k distinct columns $j_1, j_2, ..., j_k$ [2]. We calculate the sequences $S_9(2)$ by a program written in *C*. For each code, the codewords of weight 9 are obtained by the above fact used to determine the numbers $A_9, A_{10}, ..., A_{14}$. From this calculation, we know that the 487 self-dual codes have distinct sequences. Hence the 487 self-dual codes are inequivalent.

Theorem 1 *There are exactly 487 inequivalent self-dual codes over* $GF(7)$ *generated by Hadamard matrices of order 28.*

Table 1: Weight distributions of the three codes

A_0	$A_1,\ldots,A_7$	A_8	A_9	A_{10}	A_{11}	A_{12}	A_{13}
1	0	2280	0	23408	72960	241680	437760
A_{14}	A_{15}	A_{16}	A_{17}	A_{18}	A_{19}	A_{20}	
1203840	1586880	2229840	1901520	1418160	528960	118336	

Remark 1 Some codes have the same sequences $S_9(1)$.

Remark 2 Let $C'(H_i)$ be the subcode of $C(H_i)$ generated by the set of minimum weight codewords. $C'(H_{10})$ has dimension 10, $C'(H_{368})$ has dimension 12, $C'(H_{474})$ has dimension 12 and the other codes have dimension 14.

Remark 3 It is known that there are exactly three inequivalent Hadamard matrices of order 20 and these matrices generate self-dual codes over $GF(5)$ (cf. [1, p. 287, Remark (6)]). Although the three self-dual codes have the same weight distribution, these codes have distinct sequences $S_8(4)$. Hence the three self-dual codes are inequivalent. The weight distributions are calculated by MAGMA. The weight distribution is listed in Table 1. The sequences $S_8(4)$ are calculated by the same method as done for the case that $n = 28$ and $p = 7$.

Acknowledgment

The author would like to thank his supervisor Professor Masaaki Harada for his helpful suggestions, discussions and encouragement. The author also would like to thank Professor Vladimir D. Tonchev and Hiroyuki Nakasora for their helpful comments on a preliminary version of this note and Professor Takuji Nishimura for his helpful advice for computer programming.

References

[1] E. F. Assmus Jr. and J. D. Key, Designs and Their Codes, Cambridge University Press, Cambridge, 1992.

[2] M. Harada, New extremal ternary self-dual codes, Australas. J. Combin. 17 (1998), 133–145.

[3] M. Harada, On the self-dual $\mathbf{F}_5$-codes constructed from Hadamard matrices of order 24, J. Combin. Designs, (to appear).

[4] H. Kimura, Classification of Hadamard matrices of order 28, Discrete Math. 133 (1994), 171–180.

[5] C. W. H. Lam, L. Thiel and A. Pautasso, On ternary codes generated by Hadamard matrices of order 24, Congr. Numer. 89 (1992), 7–14.

[6] V. Pless, Introduction to the Theory of Error-Correcting Codes, Third edition, A Wiley-Interscience Publication, John Wiley & Sons, New York, 1998.

[7] V. S. Pless and V. D. Tonchev, Self-dual codes over $GF(7)$, IEEE Trans. Inform. Theory 33 (1987), 723–727.

[8] N. J. A. Sloane, A Library of Hadamard Matrices, available at http://www.research.stt.com/~njas/hadamard/.

A Weight Distributions of 487 Codes

In this appendix, we give the weight distributions of the 487 self-dual codes over $GF(7)$ generated by Hadamard matrices of order 28 by listing $A_9, A_{10}, ..., A_{14}$.

$C(H_i)$	A_9	A_{10}	A_{11}	A_{12}	A_{13}	A_{14}
1	252	1704	12996	82452	745740	4698828
2	312	1728	13032	82536	744624	4694976
3, 6	228	2112	13140	78396	753372	4671900
4, 5	204	2112	12852	78732	756756	4667796
7, 19	288	1680	13104	82704	743400	4700952
8, 21	252	2016	13104	79464	751212	4677372
9, 17	192	1680	12852	82548	747936	4700736
10	264	2040	12960	79560	752904	4672440
11, 14	240	1488	12852	84564	742896	4712976
12	234	1320	13050	85710	737514	4729410
13	432	1680	13320	83208	738216	4698360
15	198	1464	12726	84738	744534	4714398
16	360	1824	13104	81840	744912	4686912
18, 20	408	1824	13104	82128	743904	4684752
22	648	2400	13104	78384	750960	4630752
23, 126	324	1368	13104	85728	736092	4722732
24, 125	264	984	12600	89664	734328	4745160
25, 78	396	1320	12924	86892	735372	4719852
26, 120	234	1248	12654	87018	739962	4727682
27, 139	204	936	12384	90096	736740	4747572
28, 137	216	1008	12420	89460	737640	4742280
29, 138	330	1200	12762	87846	735858	4728906
30, 86	264	1536	12924	84156	742680	4709592
31, 181	384	1536	13212	84396	737280	4709376
32, 119	288	1632	12816	83616	745272	4699368
33, 121	312	1296	12780	86844	738072	4722840
34, 107	360	1440	12924	85596	738648	4712472
35, 108	240	1392	12780	85548	741600	4718880
36, 100	204	1824	12924	81204	749988	4690692
37, 69	264	2136	13320	78096	751320	4671720
38, 105	276	1512	13032	84264	740844	4712796
39, 104	300	1680	13104	82776	743148	4700412
40	264	1632	13104	82992	742896	4705632
41, 42	312	1632	13104	83280	741888	4703472
43	432	1392	13032	86280	735048	4714776
44	408	1536	12960	84960	739296	4703760
45, 46	360	1440	12960	85536	738288	4713120
47	312	1056	12672	89184	734112	4738896
48	600	2208	13248	79584	746496	4649904
49, 173	420	840	12852	91476	725508	4753476
50	444	1104	12852	89244	730548	4732596
51, 154	396	1104	12708	89196	732996	4732164
52, 67	420	1128	12708	89124	732996	4729284
53, 97	336	1104	12564	89076	735696	4732272

$C(H_i)$	A_9	A_{10}	A_{11}	A_{12}	A_{13}	A_{14}
54, 160	468	1344	12960	87048	734004	4715460
55, 136	210	1008	12222	89754	739746	4738986
56	378	1224	12834	87798	734634	4726242
57, 149	360	696	12096	93672	731304	4753368
58, 134	240	768	12276	92004	733536	4756608
59, 156	288	768	12276	92292	732528	4754448
60, 63	324	1296	12708	87036	738540	4721004
61	312	1152	13104	87600	731808	4739472
62, 135	192	1296	12564	86484	742752	4724352
64, 188	270	1056	12690	88902	734814	4741110
65, 82	486	1464	12798	86346	737766	4702734
66, 151	420	1440	13176	85536	734868	4714308
68, 99	168	1656	12780	82740	748656	4702320
70	378	1224	12618	88158	736794	4722354
71, 85	360	1008	12312	90504	735696	4733856
72, 83	384	1032	12420	90252	734616	4732920
73, 81	432	1392	12744	86760	737928	4709592
66, 151	420	1440	13176	85536	734868	4714308
68, 99	168	1656	12780	82740	748656	4702320
70	378	1224	12618	88158	736794	4722354
71, 85	360	1008	12312	90504	735696	4733856
72, 83	384	1032	12420	90252	734616	4732920
73, 81	432	1392	12744	86760	737928	4709592
74, 87	258	1080	12546	88854	737010	4737258
75	288	1056	12060	90060	740736	4728960
76, 95	348	912	12276	91356	734292	4740948
77, 146	228	888	12240	90912	736668	4747500
79	288	1104	12744	88488	734904	4737672
80, 106	354	1392	13158	85602	735426	4720554
84, 172	384	1104	12672	89184	733608	4732056
88, 170	432	1008	12564	90516	731664	4735152
89	714	1464	12906	87534	731898	4694418
90, 91	252	1320	12780	86268	739836	4723740
92, 159	396	1104	12816	89016	731916	4734108
93, 132	312	1200	12888	87528	734976	4731984
94	432	1104	12492	89772	734400	4726656
96, 186	468	960	12924	90564	726300	4743612
98	450	1080	12726	89706	731178	4731858
101, 165	216	1872	13068	80604	749304	4689144
102, 167	324	1560	13248	83760	738684	4710924
103, 168	252	1728	13176	81936	744444	4700268
109	372	1128	12636	88956	734724	4730148
110	462	864	12402	92262	729630	4741686
111, 177	372	1128	12672	88896	734364	4730796
112	360	1008	12744	89784	731376	4741632
113	504	1032	13032	89952	725976	4738536
114, 157	336	936	12240	91128	735408	4739040
115	330	744	12222	92850	731682	4753386
116, 118	96	1248	12024	87240	749160	4722552
117	168	1728	12528	82512	752688	4692384
122	270	888	12186	91254	736326	4744638
123	468	864	12780	91668	725724	4748220

$C(H_i)$	A_9	A_{10}	A_{11}	A_{12}	A_{13}	A_{14}
124, 190	504	864	12672	92064	726048	4744656
127, 131	234	864	12330	91014	735138	4750650
128	342	840	12582	91458	729846	4752126
129	450	1032	12762	90078	729810	4736106
130, 143	348	1032	12780	89436	731772	4741020
133, 185	360	1128	12708	88764	734256	4731984
140	228	1464	12672	85008	744444	4712076
141, 150	216	864	12240	91056	736416	4749840
142, 174	360	1176	12708	88332	735264	4728384
144, 175	360	888	12672	90984	729576	4749336
145, 171	360	1344	12816	86640	737712	4717728
147, 169	468	1272	12780	87996	734292	4717620
148, 155	366	1392	12942	86034	737334	4716126
152	432	936	12636	91044	729432	4741848
153, 166	252	1056	12564	89004	736452	4739652
158	588	1224	13104	88608	727524	4721652
161, 187	384	1176	12996	87996	731880	4732488
162	558	1680	12942	84594	739350	4685886
163, 164	192	2112	13032	78360	755208	4671576
176, 184	564	1200	13032	88800	728244	4723236
178	378	1032	12726	89706	731682	4738698
179	648	1248	13248	88512	725328	4719744
180, 192	408	1752	13392	82296	739512	4695336
182	564	1368	13500	86508	727092	4719060
183	456	1152	12528	89424	734544	4722624
189	594	576	13050	94566	714330	4769010
191	768	1032	11016	94896	740592	4690368
193	372	696	12348	93324	728532	4757364
194, 323	324	984	12636	89964	732708	4743108
195	360	1056	12528	89712	734544	4734144
196	342	1560	13014	84258	740646	4705902
197, 377	336	1176	12816	88008	734688	4731408
198, 273	276	1200	12780	87492	736812	4731660
199, 406	354	1200	12942	87690	733554	4731066
200	432	1272	13068	87300	732168	4724424
201	480	936	12096	92232	733824	4729968
202, 257	204	768	12024	92208	736812	4753692
203, 264	228	576	11952	94200	732996	4765716
204, 372	432	1056	12852	89604	729792	4736736
205, 258	366	744	12294	92946	730206	4753062
206, 312	432	912	12528	91440	730008	4741704
207, 391	252	1152	12600	88080	738108	4733100
208, 209	378	1176	12870	88170	733266	4730490
210, 461	432	720	12096	93888	730296	4748328
211, 424	552	1008	12816	90816	726624	4734288
212, 379	252	840	12312	91368	734436	4751316
213, 290	360	744	12384	92760	729432	4754952
214, 216	420	1104	12564	89580	733932	4728492
215	384	768	12024	93288	733032	4745592
217, 276	486	1272	13194	87414	729774	4724262
218, 400	312	912	12420	90900	733608	4745160
219, 420	444	1032	12600	90312	731556	4733460

$C(H_i)$	A_9	A_{10}	A_{11}	A_{12}	A_{13}	A_{14}
220, 279	324	1200	12564	88140	737964	4725612
221, 299	276	720	12348	92532	731052	4759884
222, 227	222	864	12366	90882	735030	4751838
223, 440	468	936	12960	90720	725436	4746060
224, 448	312	888	12420	91116	733104	4746960
225, 459	384	816	12564	91956	728640	4751712
226, 462	360	792	12312	92448	731160	4750056
228, 364	264	576	12096	94176	730800	4766688
229, 339	420	912	12996	90588	725580	4750668
230, 283	288	960	12528	90144	734040	4744584
231, 267	300	1224	12816	87360	736452	4729428
232, 266	300	1104	12672	88680	735372	4735836
233, 265	312	1368	12852	86076	738864	4718736
234, 235	312	1536	12528	85104	745632	4700304
236	264	1056	12240	89616	739440	4733280
237, 245	348	1200	12564	88284	737460	4724532
238	312	1560	13104	83928	740376	4708872
239, 274	240	1440	12852	84996	741888	4716576
240, 367	288	984	12276	90348	737064	4738248
241, 244	396	1032	12420	90324	734364	4732380
242, 243	492	1392	12564	87420	738468	4703652
246, 348	390	888	12726	91074	728406	4748958
247, 417	192	696	11952	92904	736272	4758336
248, 393	348	792	12420	92196	730332	4752540
249	228	1248	12564	87132	740988	4726332
250, 349	540	1152	12888	89328	729180	4725324
251, 350	756	1104	13320	90336	719316	4726980
252	600	1464	12708	87180	736272	4695984
253, 360	432	1200	12924	88188	732096	4727232
254, 359	360	1248	12564	87924	738216	4720392
255, 443	354	912	12438	91122	732546	4743594
256, 353	444	1008	12636	90468	730692	4735908
259, 414	288	576	12384	93840	727416	4770792
260, 428	312	960	12456	90408	734256	4742208
261, 263	252	720	12168	92688	733356	4757724
262, 284	240	768	12492	91644	731376	4760496
268, 275	372	1344	13032	86352	735300	4721076
269, 303	492	1128	13212	88716	726444	4735116
270, 293	372	1200	12852	87948	734076	4728636
271, 285	300	1128	12744	88344	735156	4735332
272, 335	420	1248	12888	87744	733716	4723524
277, 324	336	1008	12564	89940	733680	4739472
278	432	1464	13104	85512	735840	4710672
280	504	1320	12960	87480	732744	4715640
281, 358	372	792	12564	92100	728388	4754052
282, 433	600	984	13392	90360	719352	4744296
286, 352	294	1152	12618	88302	737046	4731534
287, 301	288	768	12384	92112	731448	4756392
288	264	648	11448	94608	738792	4749624
289	288	1608	9504	89352	777888	4641552
291	444	1320	12132	88500	742284	4703436
292, 376	240	792	12276	91788	734040	4754808

$C(H_i)$	A_9	A_{10}	A_{11}	A_{12}	A_{13}	A_{14}
294, 340	546	936	13014	91098	723258	4743522
295, 403	378	1128	12762	88782	733338	4732146
296, 430	336	1152	12816	88224	734184	4733208
297, 319	366	960	12690	90342	730782	4743990
298, 398	528	720	12744	93384	721800	4755672
300	252	1512	13104	84000	740628	4715172
302	576	1320	12888	88032	731952	4711104
304	432	1176	12924	88404	731592	4729032
305, 415	324	1056	12636	89316	734220	4737708
306, 402	444	1008	12996	89868	727092	4742388
307, 442	324	1200	12708	87900	736524	4728204
308, 407	444	864	12744	91584	726588	4748652
309, 363	276	936	12348	90588	735588	4743684
310, 334	468	1176	13104	88320	729036	4730652
311, 375, 422, 429	192	792	12168	91680	736128	4755024
313, 382	360	840	12528	91656	730008	4750344
314	600	1032	13032	90528	723960	4734216
315, 357	504	1032	12816	90312	728136	4734648
316, 362	120	720	11880	92376	739008	4758480
317, 386	498	1080	13014	89514	727290	4734882
318, 327	276	672	12060	93444	732924	4758300
320, 452	336	1056	12672	89328	733608	4737816
321, 325	324	648	12420	93348	727812	4764420
322, 351	96	792	12024	91344	739584	4756752
326	372	1464	13104	85152	737100	4713372
328, 411	474	1296	13050	87366	731970	4720410
329, 383	612	888	13068	91836	720324	4745124
330	600	1032	12456	91488	729720	4723848
331, 361	348	768	12348	92532	730548	4753044
332, 434	312	1008	12528	89856	734544	4739904
333, 388	588	840	13104	92064	719460	4750452
336, 342	360	1368	12852	86364	737856	4716576
337	288	1464	11952	86568	750384	4696416
338, 387	648	840	13140	92364	717840	4748400
341, 346	492	648	12564	94116	722844	4759452
343, 347	324	1224	13320	86664	730908	4737420
344, 453	360	1344	13320	85800	732672	4726800
345	480	1272	13248	87288	729360	4725504
354, 384	312	816	12384	91824	731952	4751712
355	366	960	12834	90102	729342	4746582
356, 435	336	1080	12744	88992	733392	4737312
365	456	456	11880	96768	726408	4763160
366	408	456	11880	96480	727416	4765320
368	360	456	11160	97392	735624	4754520
369	1080	1320	14184	88896	708408	4711752
370, 475	168	168	10584	99792	739368	4774392
371, 474	168	168	11448	98352	730728	4789944
373, 451	288	696	12276	92940	731016	4759848
374, 416	204	1032	12348	89292	739116	4739724
378, 397	444	1032	12636	90252	731196	4734108
380	768	936	13176	92160	716976	4736448
381, 426	588	768	13104	92712	717948	4755852

$C(H_i)$	A_9	A_{10}	A_{11}	A_{12}	A_{13}	A_{14}
385, 427	468	840	12780	91884	725220	4750020
389, 392	450	840	12690	91926	726498	4749210
390, 399	600	864	13176	91800	718992	4749408
394	1032	1032	13104	93000	714168	4716072
395	552	888	12528	92376	726984	4738104
396, 419	408	1176	13032	88080	731016	4732056
401	360	1032	12816	89448	731160	4741128
404	528	1320	12816	87864	733680	4711968
405	504	1320	12744	87840	734904	4711752
408, 449	312	672	12240	93360	730368	4759920
409, 460	312	912	12492	90780	732888	4746456
410	444	1176	12708	88836	733500	4724604
412, 465	240	1152	12672	87888	737640	4734936
413	408	552	12096	95256	727272	4762008
418, 464	456	888	13320	90480	721080	4756680
421	312	1032	12312	90000	737208	4734216
423, 444	384	1200	12780	88140	734544	4726800
425, 436	168	840	12600	90384	733320	4760280
431, 432	384	528	12384	94848	724392	4770072
437, 467	168	1512	13104	83496	742392	4718952
438, 469	552	1056	13032	90024	725472	4734576
439	480	1608	12384	85704	745056	4684752
441	612	1080	13212	89868	722916	4733316
445	336	1056	12816	89088	732168	4740408
446	408	936	12600	90960	730296	4742280
447	876	888	12996	93540	715500	4731948
450	1128	1032	11880	95616	724392	4689720
454, 456	408	816	12960	91440	724176	4757760
455	624	744	12240	94584	725328	4740480
457	624	1560	14040	84240	724464	4711680
458	648	648	12456	95232	720648	4750488
463, 480	216	1152	12456	88104	740304	4732128
466	528	1248	13968	86592	720648	4738104
468	816	528	12996	96420	709200	4761648
470	984	576	11952	98736	717120	4731696
471	1584	648	11664	102168	708912	4694112
472	672	456	11808	98184	722592	4752144
473	2184	2184	13104	89544	714168	4577832
476	696	168	10584	102960	728280	4750632
477	960	648	13824	94824	700416	4761072
478	0	2184	0	98280	891072	4440240
479	720	648	12960	94824	714096	4756320
481	432	960	12528	91008	731016	4738104
482, 486	168	864	12240	90768	737424	4752000
483, 484	312	1224	12708	87612	737280	4726944
485	234	936	12510	90066	734850	4748490
487	6552	0	0	157248	707616	4309200