AKCE J. Graphs. Combin., **1,** No. 1 (2004) 51-61

Graph Reconstruction - Some New Developments*

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Communicated by: S. Arumugam
Received: June 07, 2004

Abstract

The reconstruction conjecture claims that every graph on at least three vertices is uniquely determined by its collection of vertex deleted subgraphs. Even though it is one of the foremost unsolved problems in Graph Theory, work on it has slowed down, may be due to the general feeling that existing techniques are not likely to lead to a complete solution. Here we give the results that have appeared recently in some selected variations of the reconstruction problem like edge reconstruction, degree associated reconstruction, vertex switching reconstruction and reconstruction numbers.

Key words: Reconstruction, graphs, digraphs, vertex switching, reconstruction numbers.

AMS Subject Classification: 05C60

1 Introduction

Reconstruction Conjecture (RC) was first considered by Kelly and Ulam in 1941 when P. J. Kelly wrote his dissertation under S. M. Ulam. Kelly wrote the first paper on reconstruction [22] in 1957 and more than 300 papers have appeared so far on this topic. Graph isomorphism testing is another important problem in Graph Theory and no good characterization of pairs of isomorphic graphs has been found. Zemlyatshenko [62] proved that isomorphism testing for graphs with n vertices can be performed in c^n time assuming the RC. Even though this result has been superseded by the $\exp\left(c\sqrt{n \log n}\right)$ time algorithm which does not rely on RC that is given in [1], it is strongly believed that truth of RC will simplify isomorphism testing for graphs. [4] is an exhaustive survey on graph reconstruction by Bondy and Hemminger. [5, 7, 33, 37, 38, 41, 45] are some other survey articles on this topic. In general we use the terminology in Haray[15]. $\nu(G)$ and $\varepsilon(G)$ denote the number of vertices and number of edges in a graph G.

2 Vertex reconstruction

A graph H is called a *reconstruction* of a graph G if the vertices of G and H can be labeled $v_1, v_2, ..., v_n$ and $u_1, u_2, ..., u_n$ respectively such that $G - v_i \cong H - u_i$ for every i. A graph G is called *reconstructible* if every reconstruction of G is isomorphic to G. A function (parameter) f defined on the class of all graphs is called *reconstructible* if $f(G) = f(H)$ whenever H is a reconstruction of G.

Reconstruction Conjecture (RC) : *All graphs with at least three vertices are reconstructible.*

*Research Supported by Govt. of India DST Project No. MS/093/98

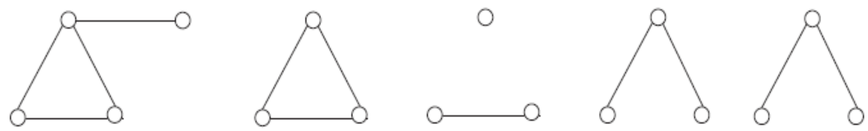

Figure 1

A vertex deleted subgraph of a graph *G* in unlabeled form is called a *card* of *G.* The collection of all cards of *G* is called the *deck of G.* A graph and its deck are shown in Figrue 1. In 1964, Harary reformulated RC as follows[14].

RC: *Any graph G with at least 3 vertices is uniquely determined up to isomorphism by its deck.*

Two approaches are adopted generally. One is to reconstruct classes of graphs in the hope that eventually enough classes will be found to include all graphs. The other approach is through the reconstruction of parameters in the hope that enough parameters to determine the graph completely will be reconstructed. The following fundamental lemma is due to Kelly.

Lemma 2.1 [22] *For any two graphs F and G such that* $v(F) < v(G)$, *the number* $s(F, G)$ *of subgraphs of G isomorphic to F is reconstructible. (Two subgraphs isomorphic to F are counted as different if they have different vertex set or edge set).*

Corollary 2.2 *For any two graphs F and G such that* $v(F) < v(G)$, *the number of subgraphs of G which are isomorphic to F and include a given vertex v is reconstructible from the deck of G. (This number is in fact* $s(F, G) - s(F, G - v)$ *).*

Obviously a graph is reconstructible if its complement is reconstructible. All graphs with at most 10 vertices are verified to be reconstructible by Mckay using computers. Classes of graphs that are proved to be reconstructible include regular graphs, disconnected graphs, trees, unicyclic graphs, cactus, maximal planar graphs, outerplanar graphs, separable graphs without endvertices, some other classes of separable graphs, some classes of blocks and neighbourhood degree sum regular graphs. Muller [39] has shown using probabilistic methods that almost all graphs are reconstructible.

Parameters proved to be reconstructible include neighbourhood degree sequence (Manvel), blocks of the graph and connectivity. Tutte has reconstructed [60] the dichromatic polynomial. The dichromatic polynomial of *G* is defined by the formulae

$$Q(G; x, y) = \sum_{F \subseteq G} x^{\omega(F)} \cdot y^{\varepsilon(F) - v(F) + \omega(F)}$$

where the sum is taken over all spanning subgraphs *F* of *G* and $\omega(F)$ denotes the number of components of *F* . Since chromatic polynomial

$$P(G, x) = (-1)^{v} Q(G; -x, -1),$$

chromatic polynomial and hence chromatic number of graph are also recostructible. In addition, the dichromatic polynomial yields three new reconstructible parameters.

(1) The number of disconnected spanning subgraphs of G having a specified number of components in each isomorphism class. (Thus, number of perfect matchings is reconstructible)
(2) The number of connected separable spanning subgraphs of G having a specified number of blocks in each isomorphism class. (Thus, number of spanning trees is reconstructile).
(3) The number of nonseparable spanning subgraphs with a given number of edges. (Thus, number of Hamilton cycles is reconstructible).

Kelly's lemma provides information on vertex proper subgraphs. The above three results give information about spanning subgraphs. Pouzet [46] has proved that characteristic polynomial is reconstructible if and only if number of Hamilton cycles is reconstructible. Hence characteristic polynomial and number of Hamilton cycles are reconstructible.

Kocay [23, 24] has reconstructed bidegreed graphs with two vertices of degree k and all others of degree $k-1$ for $k=3$. He also reported some progress with the case $k=4$ by introducing a method for extending partial automorphisms. In 1988 Yang Yongzhi has proved [61] that RC is true if all 2-connected graphs are reconstructible. Krishnamoorthy and Parthasarathy in 1976 has proved [30] that critical blocks are reconstructible. Hence RC is true if all blocks having a verex v such that $G-v$ is also a block are reconstructible.

3 Edge reconstruction

Instead of vertex deleted subgraphs, edge deleted subgraphs are used to reconstruct the graph.

Edge Reconstruction Conjecture [14] : *All graphs with at least four edges are reconstructible from the collection of edge deleted subgraphs (often called edge deck).*

The pair of graphs given in Figure 2 and Figure 3 are not edge reconstructible.

Lemma 3.1 *For any two graphs F and G such that $\varepsilon(F) < \varepsilon(G)$, the number of subgraphs of G isomorphic to F is edge reconstructible.*

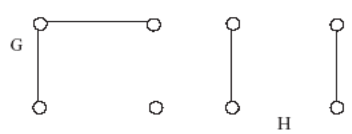

Figure 2

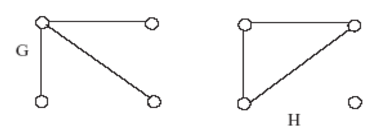

Figure 3

Theorem 3.2 (Hemminger 1969) : *A graph is edge reconstructible if and only if its edge graph is reconstructible.*

Greenwell in 1971 showed that for a graph without isolated vertices, reconstructibility implies edge reconstructibility by proving the following.

Theorem 3.3 [13] *The vertex deck of a graph G with at least four edges and no isolated vertices can be determined from its edge deck.*

(The graph $K_{1,3}$ is reconstructible, but not edge reconstructible).

Combining this theorem with results for vertex reconstruction, we immediately deduce that several classes of graphs and several parameters are edge reconstructible. Using the inclusion exclusion principle in an ingenious way, Lovasz proved the following.

Theorem 3.4 [34] *G is edge reconstructible if* $\varepsilon(G) > \binom{v}{2}$.

Muller improved this result as follows.

Theorem 3.5 [40] *G is edge reconstructible if* $2^{\varepsilon-1} > v!$.

Nash-Williams proved that graphs that are not edge reconstructible satisfy a very restrictive condition.

Theorem 3.6 (Nash-Williams Lemma) [45]: *If a graph G is not edge reconstructible then for all* $A \subseteq E(G)$, $A \equiv |E(G)| (\bmod 2)$, *there is a permutation* β *of* $V(G)$ *such that* $E(G) \cap \{u\beta v\beta \mid uv \in E(G)\} = A.$.

This theorem implies Muller's theorem. Other applications of this theorem can be seen in [5, 8, 28, 47]. However, Pyber has proved the following theorem, which he calls a 'somewhat disappointing result'.

Theorem 3.7 [48] *For* $v \geq v_0$ *there exists a (bipartite) graph* G_v *satisfying the condition of the Nash-Williams lemma with* $n > v$ *vertices and* $\varepsilon \geq n\left(\sqrt{\frac{1}{2}\log_2 n}\right)$ *edges.*

Some other important results on edge reconstruction have also been proved and [7, 32, 37] are surveys of it. Lauri has proved [31] the edge reconstructibility of planar graphs with minimum degree 5. Fiorini and Lauri have edge reconstructed [10] 4-connected planar graphs of minimum degree 4. They have also proved [11] that 3-connected graphs which triangulate a surface are edge reconstructible. It has been proved by Myrvold, Ellingham and Hoffman [44] that bidegreed graphs are edge reconstructible. They have also shown that all graphs that do not have three consecutive integers in their degree sequence are edge reconstructible.

A *claw* is an induced subgraph isomorphic to $K_{1,3}$. Ellingham, Pyber and Xingxing Yu [8] and Thatte [59] have proved that claw-free graphs are edge reconstructible. (This includes line graphs as a special case). Krasikov considered the wider class of $K_{l,m}$-free graphs, where $m \geq 3$ and proved the following.

Theorem 3.8 [28] *A* $K_{l,m}$ *-free graph G is edge reconstructible if* $\Delta(G) > cm\sqrt{\log m}$ *where c is an appropriate constant.*

Pyber has edge reconstructed [47] hamiltonian graphs of sufficiently high order. A chordless cycle is an induced subgraph isomorphic to a cycle of length four or more and a chordal graph is a graph having no chordless cycles. Thatte has proved the edge reconstructibility of chordal graphs [59] in 1995.

4 Reconstruction of digraphs

The problem of reconstructing digraphs from vertex deleted subdigraphs was first raised by Harary [14] in 1964. It developed into the following conjecture in Manvel [36] in 1974.

Digraph Reconstruction Conjecture (DRC) : *Any digraph D with at least seven vertices can be reconstructed from its subdigraphs* $D_i = D - v_i$.

The exclusion of digraphs with less than seven vertices was necessitated by pairs of nonreconstructible digraphs on 6 vertices discovered by Beineke and Parker [3]. Figure 4 contains three pairs of nonreconstructible digraphs, respectively on 3,4 and 6 vertices.

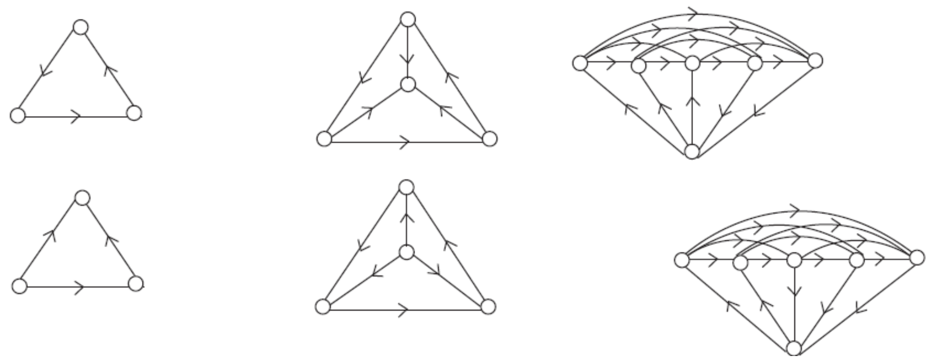

Figure 4: Three pairs of non reconstructible digraphs

The existing theorems on digraph reconstruction are rather limited. Harary and Palmer [18] have proved that nonstrong tournaments are reconstructible. Manvel [35] has shown that the degree pair sequence of any digraph *D* with five or more vertices can be derived from its subdigraphs D_i. Gnanvo and Ille [12] have proved that tournaments with at least 7 vertices and without diamonds (the two digraphs on 4 vertices in Figure 4) are reconstructible.

Stockmeyer conducted computer search for counterexamples to DRC and in 1976 announced [57] that for each integer $p \geq 5$ of the form $p = 2^m + 2^n$ with $0 \leq m < n$, there exist six related pairs of counterexamples to DRC on *p* vertices including a pair of tournaments. These are in fact extensions of the tournament counterexamples given by him for all orders of the form $2^n + 1$ and $2^n + 2$ in [56]. While concluding this paper, Stockmeyer observed, "Perhaps the considerable effort being spent in attempts to prove the (reconstruction) conjecture should be balanced by more serious attempts to construct counterexamples", summarizing the opinion prevalent at that time.

When he was working on a test to decide whether there exists a graph (digraph) having a given collection of graphs (digraphs) as its deck [49], Ramachandran observed that the existing counterexamples to DRC did not behave with the test in the same way as a counterexample to RC was expected to behave. This showed that the extension of the test from graphs to digraphs differed from the way in which RC was extended to digraphs and led to a new type of digraph reconstruction in [50].

In a digraph, if uv and vu are both arcs, then they together are called a *symmetric pair* of arcs. If uv is an arc and vu is not an arc, then uv is called an *unpaired outarc* incident with *u* and an *unpaired inarc* incident with *v*. For a vertex *v* of a digraph, the ordered triple (r, s, t) is called the *degree triple* of *v* where *r, s* and *t* are respectively the number of unpaired outarcs, unpaired inarcs and symmetric pairs of arcs incident with *v*. Clearly $r+t$ and $s+t$ are respectively the outdegree and indegree of *v*. The ordered pair $\left(d(v), G-v\right)$ is called a *degree assocoiated card* or *dacard* of a graph (digraph) *G* where $d(v)$ is the degree (degree triple) of *v* in *G*. The *degree associated deck* or *dadeck* of a graph (digraph) *G* is its collection of dacards. A digraph is called *N-reconstructible* [50] if it can be determined uniquely from its dadeck.

New Digraph Reconstruction Conjecture (NDRC) [50] : *All digraphs are N-reconstructible.*

It is well known that the dadeck of a graph having at least three vertices can be determined from its deck. Moreover, graphs can be considered as a subclass of digraphs in which arcs always occur as symmetric pairs. Hence NDRC reduces to RC in the case of graphs and the truth of NDRC implies the truth of RC. Thus NDRC is stronger than RC, but weaker than the disproved DRC. In [50, 51, 52, 54] some classes of digraphs are proved to be *N*-reconstructible and all the known counterexamples to DRC including those [58] found in 1991 belong to these classes. Thus the threat posed to RC by the wave of counterexamples to DRC has been removed, as NDRC which is stronger than RC withstands them.

Manvel has suggested [37] splitting of NDRC into reasonable parts as the following.

Conjecture [37] : *A digraph is N-reconstructible if the underlying graph is reconstructible.*

5 Extension to Hypergraphs and Unification

A *k*-hypergraph *G* consists of a vertex set $V(G)$ and an edge set $E(G)$, a set of *k*-subsets of $V(G)$. If $X \subseteq V(G)$, the edges of the induced subhypergraph $G[X]$ are those edges of *G* whose vertices are all contained in *X*. If $v \in V(G)$, then $G-v$ denotes the induced subhypergraph $G\left[V(G)-\{v\}\right]$. *Reconstruction* and *reconstructibility* of a hypergraph are defined as in the case of graphs. Kocay [25] and Kocay and Lui [26] have constructed a family of nonreconstructible 3-hypergraphs with $2^n + 2^m$ vertices, for all $n, m \geq 1$.

Hashiguchi [21] has extended RC to hypergraphs as Double Reconstruction Conjecture and shown that none of the counterexamples in Kocay [25] satisfy the hypothesis of the double reconstruction conjecture. In his attempts to show that the nonreconstructible hypergraphs so far known need not be taken as a threat to RC, Ramachandran has unified the reconstruction problem as follows [53].

For a vertex *v* of a hypergraph, let $N(v) = \{v\} \cup \{w \mid$ there exists an edge incident with both *w* and $v\}$ and $E(v)$ be the set of edges incident with *v*. The subhypergraph with vertex set $N(v)$, edge set $E(v)$ and rooted at *v* is called the *star* at *v* and is denoted as $ST(v)$.

Unified Reconstruction Conjecture (URC) [53] : *If G and H are both graphs, digraphs or hypergraphs whose vertices can be labeled* $v_1, v_2, ..., v_n$ *and* $w_1, w_2, ..., w_n$ *such that* $\left(ST(v_i), G-v_i\right) \cong \left(ST(w_i), H-w_i\right)$ for $i = 1,2,...,n$ then $G \cong H$.

This conjecture reduces to RC in the case of graphs and NDRC in the case of digraphs. It is proved [53] that none of the nonreconstructible hypergraphs in Kocay [25] is a counterexample to URC.

We observe that the digraph and hypergraph analogues of *degree of a vertex v in a graph* is *star at v* and the information on star at *v* may have to be used while trying to settle RC as it is the single concept with which the disproved DRC and the analogous Hypergraph Reconstruction Conjecture have been salvaged, still as extensions of RC.

6 Vertex Switching Reconstruction

A *vertex switching* G_v of a graph *G* is obtained by taking a vertex *v* of *G*, removing all edges incident with *v*, and adding edges joining *v* to every vertex not adjacent to *v* in *G*. The multiset $< G_v : v \in V(G) >$ of unlabeled graphs is called the *vertex switching deck* of *G*. See Figure 5. A graph *G* is called *vertex switching reconstructible* if any graph with the same vertex switching deck as *G* is isomorphic to *G*. Vertex switching reconstruction was first considered by Stanley [55] in 1985 and some good results have been proved.

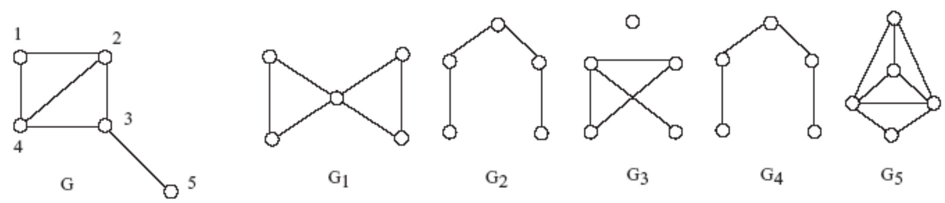

Figure5: A graph *G* and its vertex switching deck

Theorem 6.1 [55] *If* G *has n vertices and* $n \not\equiv 0 \pmod 4$, *then G is vertex switching reconstructible.*

Theorem 6.2 [55] *When* $n \neq 4$, *number of edges and degree sequence are vertex switching reconstructible.*

Not all graphs are vertex switching reconstructible. $4K_1$ and C_4 are a pair of nonisomorphic graphs having the same vertex switching deck. There are some other such examples, all on 4 vertices.

Theorem 6.3 [27] *Disconnected graphs of order* $n \neq 4$ *are vertex switching reconstructible.*

Theorem 6.4 [9] *Triangle free graphs are vertex switching reconstructible.*

Theorem 6.5 [9] *Regular graphs of order* $n \neq 4$ *are vertex switching reconstructible.*

A vertex switching analogue of Kelly's Lemma (Lemma 2.1 above) is proved in [29, 9]. However, solving a system of linear equations is involved in it.

For $W \subset V, G_W$ is the graph obtained from G by replacing all edges between W and $G - W$ by nonedges and vice versa. The *multiset* $< G_W : |W| = s >$ is called s-switching deck of G. s-switching reconstructibility was introduced in [55] and Krasikov and Roditty [29] have proved some interesting results including the following.

Theorem 6.6 [29] *A graph on n vertices is s-vertex switching reconstructible if*

(i) $s = 1$ and $n \not\equiv 0 \pmod 4$

(ii) $s = 2$ and $n \neq t^2$, where $t \equiv 0, 1 \pmod 4$

(iii) $s = 3$ and $n \not\equiv 0 \pmod 4$, $n \neq (t^2 + 2)/3$, where $t = 1,2,5,10 \pmod{12}$

Also for each $s \geq 4$, *there exists an integer* N_s *such that a graph is s-switching reconstructible provided* $n > N_s$ *when* s *is even and* $n > N_s$, $n \not\equiv 0 (\mathrm{mod}\ 4)$, *when* s *is odd.*

7 Reconstruction Numbers

For a reconstructible graph *G*, Harary and Plantholt [19] have defined the *reconstruction number* $rn(G)$ to be the minimum number of vertex deleted subgraphs in the deck of *G* which are not contained in the deck of any other graph, thus uniquely identifying the graph *G*. Thus $rn(G)$ is the size of the smallest subcollection of the deck of *G* which is not contained in the deck of any other graph $H, H \ncong G$. An *s-blocking set* of *G* is a family *F* of graphs such that $G \notin F$ and each collection of *s* cards of *G* will also appear in the deck of some graph of F. Myrvold [43] has proved that the reconstruction number of trees is 3. It has been proved (Bollobas [6], Myrvold [41]) that almost every graph has reconstruction number 3. In the case of disconnected graphs, Myrvold has proved the following theorems.

Theorem 7.1 [42] *If G is a disconnected graph with at least two different component orders, then* $rn(G) = 3$.

Theorem 7.2 [42] *If G is a disconnected graph consisting of k components of order c and the components are not all isomorphic, then* $rn(G) = 3$.

Theorem 7.3 [42] $rn(tG) \leq m + 2$ *for* $t \geq 2$ *where* G *is a connected graph of order m.*

There are graphs with arbitrarily high $rn(G)$ as seen from the following theorem.

Theorem 7.4 [19] *The graph* $2K_n$ *has reconstruction number* $n + 2$ *for* $n \geq 2$ *with* $(n+1)$-*blocking set consisting of the single graph* $K_{n+1} \cup K_{n-1}$.

Harary and Plantholt have made the following conjectures.

Conjecture [19] : *For any graph G on n vertices,*

$$rn(G) \leq \frac{n}{2} + 2$$

and equality holds if and only if G is P_4 *or* $2K_c$ *or* $K_{c,c}$.

Conjecture [19] : *If G is a graph of odd prime order, then* $rn(G) = 3$.

The following weakening of the problem has also been considered [19]. Let C be a class of graphs and let $G \in C$. *Class reconstruction number* $Crn(G)$ is defined to be the smallest number of cards of G which can determine G, given that $G \in C$. That is, $Crn(G)$ is the size of the smallest subcollection of the deck of G which is not contained in the deck of any other graph H, $H \ncong G$, $H \in C$. As an example, $Crn(G) = 1$ where C is the class of regular graphs and $G \in C$. Class reconstruction numbers for maximal planner graphs, trees, total graphs and unicyclic graphs have been studied [2, 16, 17, 20]. *Edge reconstruction number* and *class edge reconstruction number* of a graph have also been studied [33]. [33, 41] are surveys on reconstruction numbers.

$rn(G)$ is expected to serve as a measure of the level of difficulty of reconstructing G. It is possible that someone investigating these may have new insights into reconstruction conjecture itself.

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