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Domination in Hypergraphs

B. D. Acharya
Department of Science and Technology
Technology Bhawan, New Mehrauli Road
New Delhi-110 016, India.
E-mail: *bdacharya@yahoo.com*

Abstract

In this article, the notion of domination in hypergraphs is introduced as a natural extension of the notion from the theory of graphs; extensions of many basic results from the theory of domination in graphs, including the well known characterization of the minimal dominating sets due to Ore [12], to the theory of hypergraphs are then obtained.

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1. Foundations

By a *hypergraph* we mean an ordered pair $H = (X, \mathcal{E})$ where X is a finite nonempty set and $\mathcal{E}$ is a subset of the power set 2^X of X, viz., the set of all subsets of X such that

(i) $E \in \mathcal{E} \Rightarrow E \neq \emptyset$, and

(ii) $\bigcup_{E \in \mathcal{E}} E = X$.

The elements of X are called *vertices* of H and those of $\mathcal{E}$ *edges* of H.

For all terminology and notation in hypergraph theory not specifically defined here we refer the reader to C. Berge ([4], [5]).

Given a hypergraph $H = (X, \mathcal{E})$ and a nonempty subset $S \subseteq X$, the hypergraph $H_S = (S, \{E \in \mathcal{E} : E \subseteq S\})$ is called the *subhypergraph induced by* S *in* H. Also, we let $q(S) = |\{E \in \mathcal{E} : |E \cap S| \geq 2\}|$ where $|A|$ denotes the cardinality of the set A. Two vertices u and v of H are *adjacent* in H if there exists an edge of H that contains both u and v; *nonadjacent* if they are not adjacent. The set S is *stable* if it does not contain any edge E with $|E| > 1$ and the *stability number* $\alpha(H)$ of H is defined as the maximum cardinality of a stable set in H. On the other extreme is the notion of a *strongly stable set* S in which no two vertices are allowed to be adjacent; in standard graph theory literature, the term 'independent set' is used to mean a strongly stable set and hence we will use the same term for a strongly stable set in a hypergraph. The maximum cardinality of an independent set in H is called its *independence number* and is denoted $\beta(H)$. Clearly, since the set $\mathcal{S}_\alpha(H)$ of all maximal stable sets in H contains the set $\mathcal{S}_\beta(H)$ of all maximal independent sets in H it follows that

$$\alpha(H) \geq \beta(H) \tag{1}$$

for any hypergraph; also, if H is a graph then equality holds in (1).

Problem 1.1. *Characterise in general the class of hypergraphs that satisfy the equality in (1).*

Consider $D \in \mathcal{S}_\alpha(H)$. Then, for any $u \in X - D$ there must exist $v \in D$ such that $u, v \in E$ for some $E \in \mathcal{E}$, for otherwise $D \bigcup \{v\}$ would be a stable set, contrary to the maximality of D. If H is a graph, this property is known as *domination*; thus, one is naturally motivated to study this notion for hypergraphs in general.

Definition 1.2. *Let $H = (X, \mathcal{E})$ be any hypergraph. Then, $D \subseteq X$ is a dominating set of H if for every $v \in X - D$ there exists $u \in D$ such that u and v are adjacent in H; that is, if there exists $E \in \mathcal{E}$ such that $u, v \in E$.*

Lemma 1.3. *In any hypergraph $H = (X, \mathcal{E})$, the property $\mathcal{D}(H)$ of the subsets of X being dominating sets is 'superhereditary' in the sense that it satisfies the condition that*

$$\{A \in \mathcal{D}(H)\} \bigwedge \{A \subset B\} \Rightarrow B \in \mathcal{D}(H). \tag{2}$$

Proof. Let $D \subseteq X$ be such that $D \in \mathcal{D}(H)$ and let $D \subset C \subseteq X$. Suppose $C \notin \mathcal{D}(H)$. Then, there must exist, by definition, $y \in X - C$ such that no vertex in C is adjacent to y. However, since $D \subset C$ this ought to be false and hence the result follows by contraposition. □

The following is a generalisation of a result for graphs due to Berge [4].

Theorem 1.4. *Let $H = (X, \mathcal{E})$ be any hypergraph. Then every maximal stable set of H is a minimal dominating set and, conversely, every stable dominating set of H is a maximal independent set in H.*

Proof. As *necessity* part of the proof is straightforward, we prove only the sufficiency part of it. Toward this end, let D be a stable dominating set of H. If D is not a maximal independent set of H then, by Lemma 1.3, it follows that there must exist $x \in X - D$ such that $D \cup \{x\}$ is independent in H whence x is not adjacent to any vertex in D, a contradiction to the hypothesis that D is a dominating set of H. Thus, the result follows by contraposition. □

Corollary 1.5. *In any hypergraph* $H = (X, \mathcal{E})$, *every independent dominating set is a minimal dominating set of* H.

Observation 1.6. *However, not every minimal dominating set of* H *need be independent: for example, in the 4-cycle* $C_4 = (x_1, x_2, x_3, x_4, x_1)$ *every edge* $x_i x_{i+1}$ *is a minimal dominating set.*

Problem 1.7. *Characterise in general hypergraphs in which every edge is a minimal dominating set.*

Observation 1.8. *In fact, every 2-subset of vertices of a 4-cycle is a minimal dominating set in it.*

Definition 1.9. *The minimum (maximum) cardinality of a minimal dominating set in a hypergraph* H *is called its* domination *(*upper domination*)* number *and is denoted* $\gamma(H)$ *(*$\Gamma(H)$ *respectively).*

Problem 1.10. *Characterise in general hypergraphs* $H = (X, \mathcal{E})$ *for which there exists an integer* k, $\gamma(H) \leq k \leq \Gamma(H)$, *such that every* k*-subset of* X *is a minimal dominating set of* H.

Given any vertex y in a hypergraph $H = (X, \mathcal{E})$, the set $N[y] = \{x \in X : x, y \in E$ for some $E \in \mathcal{E}\}$ is called the *neighbourhood* of y in H and each vertex in the set is called a *neighbour* of y; note that, since $\bigcup_{E \in \mathcal{E}} E = X$ by the definition of a hypergraph, it follows that $y \in N[y]$. Hence, we call $N(y) = N[y] - \{y\}$ the *open neighbourhood* of y. We can now give a characteerisation of minimal dominating sets in a hypergraph.

Theorem 1.11. *Let* $H = (X, \mathcal{E})$ *be any hypergraph and* $D \subseteq X$ *be a dominating set. Then,* D *is a minimal dominating set of* H *if and only if for every* $d \in D$ *there exists* $v \in X$ *such that*

$$N[v] \cap D = \{d\}. \tag{3}$$

Proof. Let $\mathcal{D}^m(H)$ denote the set of all minimal dominating sets of H, $D \in \mathcal{D}^m(H)$ and $d \in D$. Since $D \in \mathcal{D}^m(H)$, no proper subset of D is dominating; in particular, therefore, $D - \{d\}$ is not a dominating set of H. Hence, there must exist $v \in X - (D - \{d\})$ such that $N[v] \cap (D - \{d\}) = \emptyset$. If $v = d$, we see that $N[v] \cap D = \{v\} = \{d\}$. On the other hand, if $v \neq d$, then $v \in X - D$ and since $D \in \mathcal{D}(H)$ there must exist $u \in D$ such that u and v are adjacent. But, since $N[v] \cap (D - \{d\}) = \emptyset$, it follows that $u = d$ and hence (3) must hold.

Conversely, suppose $D \subseteq X$ satisfies the condition stated in the theorem. Then, the given condition implies that for every $d \in D$, $D - \{d\} \notin \mathcal{D}(H)$. Now, if D contains a proper dominating set D' then, there exists $x \in D - D'$ whence $D' \subset D - \{x\}$ whence by virtue of Lemma 1.3 we see that $D - \{x\} \in \mathcal{D}(H)$, a contradiction to the above derivation. The proof is then seen to be complete. □

Corollary 1.12. [11] *In any graph* $G = (V, E)$, *a dominating set* D *of its vertices forms a minimal dominating set if and only if for every* $d \in D$, *either* $N(d) \cap D = \emptyset$ *or there exists* $v \in X - D$ *such that* $N(v) \cap D = \{d\}$.

Corollary 1.13. *In any hypergraph* $H = (X, \mathcal{E})$, *a set* D *of its vertices is an independent dominating set of* H *if and only if* $|N[v] \cap D| = 1 \; \forall v \in X$.

Corollary 1.13 suggests the following special notions of domination and independence in hypergraphs, which we shall deal in the next section.

Definition 1.14. [1] *Let* $H = (X, \mathcal{E})$ *be any hypergraph and* k *be any positive integer. Then, we call* $D \subset X$ *a* k-independent *set of* H *if*

$$(E \in \mathcal{E}) \bigwedge (|E| > k) \Rightarrow |E \cap (X - D)| \geq k. \tag{4}$$

Also D *is called a* k-dominating set *of* H *if it satisfies*

$$|N[v] \cap D| \geq k \; \forall v \in X - D \tag{5}$$

and D *is called a* k-full set *of* H *if*

$$|N[v] \cap (X - D)| \geq k \; \forall v \in D. \tag{6}$$

Problem 1.15. *Prove or disprove the following statement: In any hypergraph* $H = (X, \mathcal{E})$, *a set* D *of its vertices is a* k*-independent and* k*-dominating in* H *if and only if* $|N[v] \cap D| = k \; \forall v \in X$.

2. t-dominating, t-independent and t-full sets

Main emphasis of this section will be on the parameters related to the notion of domination in hypergraphs and in obtaining good bounds for them. The following observation is crucial towards this end.

Observation 2.1. [1] *In any hypergraph* $H = (X, \mathcal{E})$, *given any positive integer* t, *a set* S *of its vertices is* t*-full if and only if* $X - S$ *is* t*-dominating.*

Theorem 2.2. [1] *In any hypergraph* $H = (X, \mathcal{E})$, *given any positive integer* t, *every* t*-independent set is* t*-full.*

Proof. Let S be a t-independent set of H. Clearly, $E \subseteq N(x) \; \forall E \in \mathcal{E}_x = \{E \in \mathcal{E} : x \in E\}$. If $E \subseteq N(x)$ and $|E| \leq t$ then there is nothing to prove. Hence, let $E \subseteq N(x)$ and $|E| > t$. Then, since $E \cap (X - S) \subseteq N(x) \cap (X - S)$, we get $t \leq |E \cap (X - S)| \leq |N(x) \cap (X - S)|$. Since the foregoing argument holds for each $x \in S$ in particular, it follows that S must be t-full in H. □

The converse of Theorem 2.2 is not true. Take, for example, the graph $G = (V, E)$ where $V = \{1,2,3,4,5,6\}$ and

$$E = \{\{1,2\},\{2,3\},\{2,6\},\{3,4\},\{3,5\},\{4,5\},\{4,6\},\{5,6\}\}.$$

Then, the set $S = \{2,4,5\}$ is a 1-full (or just *full*) set of G since $N[2] \cap (X - S) = \{1,3,6\}$, $N[4] \cap (X - S) = \{3,6\}$ and $N[5] \cap (X - S) = \{3,6\}$ but clearly is not a 1-independent set of G due to the edge $\{4,5\}$ in the subgraph $\langle S \rangle$ *induced by* S in G. Also, notice that S is 2-full but not 2-independent due to the fact that $|N[1] \cap S| = 1$.

Given a hypergraph H, the minimum (maximum) cardinality of a minimal t-dominating set in it is called the *t-domination number* (*upper t-domination number*) of H, denoted $\gamma_t(H)(\Gamma_t(H))$. Next, the maximum cardinality of a maximal t-independent (t-full) set of H is called the *t-independence* (*t-full*) *number* of H, denoted $\beta_{0t}(H)$ ($\zeta_{0t}(H)$); the minimum cardinality of a maximal t-independent (t-full) set of H is called the *lower t-independence* (*t-full*) *number* of H, denoted $\beta_{\ell t}(H)$ ($\zeta_{\ell t}(H)$). It $t = 1$ in these terms and definitions then it will be omitted from them.

Corollary 2.3. [1] *For any hypergraph $H = (X, \mathcal{E})$, and for any positive integer t,*

(i) $\beta_{0t}(H) \leq \zeta_{0t}(H)$

(ii) $\gamma_{0t}(H) \leq |X| - \beta_{0t}(H)$.

In any hypergraph $H = (X, \mathcal{E})$, given any vertex x, $|N(x)|$ is called its *vertex-degree* and is denoted $d(x)$; then $\delta := \delta(H)$ and $\Delta := \Delta(H)$ will denote, respectively, the minimum and the maximum of the vertex-degrees in H.

Theorem 2.4. [1] *For any hypergraph $H = (X, \mathcal{E})$, and for any positive integer t,*

$$\left(\frac{t}{t+\Delta}|X|\right) \leq \gamma_t(H). \tag{7}$$

Proof. Observe that for any nonempty set D of vertices in H one has, in general, the identity

$$\sum_{x \in D} |N(x) \cap (X - D)| = \sum_{y \in X - D} |N(y) \cap D|. \tag{8}$$

In particular, therefore, if D is a t-dominating set of H then we get from (8)

$$|D| \geq t\,|X - D| \tag{9}$$

and, further, if D is a minimum t-dominating set of H (*i.e.,* if $|D| = \gamma_t(H)$) then (9) yields (7). □

If $\delta(H) \geq t$, we may get an upper bound for $\gamma_t(H)$ which has a form similar to the lower bound given by (7) above. This is shown in our next result.

Theorem 2.5. [1] *If* $H = (X, \mathcal{E})$ *is a hypergraph with* $\delta(H) \geq t$ *and* t *is any positive integer, then*

$$\gamma_t(H) \leq \left(\frac{t}{t+1}|X|\right). \tag{10}$$

Proof. If $t = 1$, this is a well known result (*cf.:* Ore [12]). Hence, let $t \geq 2$. Suppose that the result is false and $H = (X, \mathcal{E})$ is a counterexample hypergraph having the least *size* (*i.e.*, number of edges). If $r = |X| - \gamma_t(H)$ then, by our assumption, it follows that $\gamma_t(H) > \frac{t|X|}{t+1} = \frac{t}{t+1}(r + \gamma_t(H))$ from which one can deduce $\gamma_t(H) > tr$.

By the size minimality of H, the set $S = \{x \in X : d(x) > t\}$ is either empty or t-independent. Let T be a maximal t-independent set of H containing S. Then, by virtue of Observation 2.1 and Theorem 2.2, $X - T$ is t-dominating in H so that

$$|X - T| \geq \gamma_t(H) \geq rt + 1. \tag{11}$$

By the maximality of T, each $y \in X - T$ must be contained in an edge E that intersects T. Since $d(y) = t$, we get

$$|N(y) \cap (X - T)| \leq t \ \forall y \in X - T. \tag{12}$$

We now construct a sequence of distinct vertices $y_1, y_2, \ldots, y_{r+1}$ from $X - T$ as follows:

(i) y_1 is an arbitrary vertex in $X - T$;

(ii) for $1 \leq i \leq r$, y_{i+1} is an arbitrary vertex in $(X - T) - \bigcup_{j=1}^{i} N(y_j)$.

This construction is possible since from (8) and (9), we get

$$|(X - T) - \bigcup_{j=1}^{i} N(y_j)| \geq tr + 1 - it = t(r - i) + 1. \tag{13}$$

The resulting set $U = \{y_1, y_2, \ldots, y_{r+1}\}$ is t-independent by construction. Therefore, $X - U$ must be a t-dominating set. This set has cardinality $|X| - (r + 1) < \gamma_t(H)$, a contradiction to the minimality of $\gamma_t(H)$. □

Corollary 2.6. [1] *For any hypergraph* $H = (X, \mathcal{E})$ *and for any positive integer* t *such that* $\delta(H) \geq t$,

$$\frac{|X|}{t+1} \leq \zeta_t(H). \tag{14}$$

3. Domination-perfect hypergraphs

In this section, we will introduce the notion of domination-perfect hypergraphs after extending the well-known Allan-Laskar theorem for graphs to the realm of hypergraphs. We then raise several interesting open problems on hypergraphs that satisfy the Allan-Laskar equality: $\gamma(H) = \gamma_i(H)$.

Given a hypergraph $H = (X, \mathcal{E})$ and $S \subseteq X$ let

$$ED(S) = \{v \in X - S : |N(v) \cap S| = 1\},$$

the set of vertices in $X - S$ each of which is *efficiently dominated by* S (terminology borrowed from the theory of domination in graphs; *cf.*: Haynes et al. [9]). On the other hand, if $u \in S$, let

$$N_0(u, S) = \{v \in X - S : N(v) \cap S = \{u\}\}.$$

We shall call u a *domination-critical* vertex of S if $N_0(u, S) \neq \emptyset$. This definition is motivated from the generalization of Ore's characterization of minimal dominating sets in a graph (*see* Corollary 1.12 above).

One can first verify that

$$ED(S) = \bigcup_{u \in S} N_0(u, S).$$

Next, following a clue from Ore's characterization of minimal dominating sets in a graph (*cf.*: Ore [12]), I felt that the property $\mathcal{P} : 2^X \to \{0, 1\}$ of the subsets of X defined by saying

$$(S \neq \emptyset) \bigwedge (d \in S) \bigwedge (N(d) \cap S \neq \emptyset) \Rightarrow N_0(d, S) \textit{ is a clique} \tag{15}$$

where by a *clique* we mean any set of vertices every two of which are adjacent in H, must be related to some subtle notions in the theory of domination in hypergraphs, this being so in the case of graphs as demonstrated by Acharya and Gupta [3]. Let $\mathcal{P}(H)$ the set of all subsets of $X(H)$ that satisfy $\mathcal{P}$.

By a (*strong*) t*-claw* in H, we mean a set of t edges $E_1, E_2, \ldots, E_t$, $t \geq 3$, such that $\bigcap_{i=1}^{t} E_i \neq \emptyset$ and there exist vertices $a_i \in E_i - \bigcap_{j=1}^{t} E_j$ such that $\{a_1, a_2, \ldots, a_t\}$ is a (strongly) stable set in H.

Theorem 3.1. *If $H = (X, \mathcal{E})$ is a strong claw-free hypergraph then*

$$\mathcal{D}^m(H) \subseteq \mathcal{P}(H).$$

Proof. Suppose there exists $D \in \mathcal{D}^m(H)$ such that $D \not\subseteq \mathcal{P}(H)$. Then, by the definition of $\mathcal{P}$ there exists $v \in D$ such that $N(v) \cap D \neq \emptyset$ and $N_0(v, D)$ is not a clique. If $N_0(v, D) = \emptyset$ then, either $D = X$ or $N(x) \cap D \neq \{v\}$, $\forall x \in X - D$. Since any maximal independent set of H is a dominating set of H and $N_0(v, D)$ is not a clique, it follows that the statement $D = X$ must be false. Therefore, $N(x) \cap D \neq \{v\}$, $\forall x \in X - D$. This is again false due to the fact that since $D \in \mathcal{D}^m(H)$, by the characterization of minimal dominating sets in a hypergraph (*cf.*: Theorem 1.11), there must exist $y \in X - D$ such that $N(y) \cap D = \{v\}$. Thus, we see that $N_0(v, D) \neq \emptyset$. But then, since $N_0(v, D)$ is not a clique there must exist at least two nonadjacent vertices $u, w \in N_0(v, D)$. Also, since $N(v) \cap D \neq \emptyset$ there must exist $z \in N(v) \cap D$. Since $u, w \in N_0(v, D)$ neither u nor w can be adjacent to z. Hence, let E_1, E_2 and E_3 be any three edges of H such that $v, z \in E_1$, $v, u \in E_2$ and $v, w \in E_3$. Then, the edges E_1, E_2 and E_3 induce a strong 3-claw in H, contrary to the hypothesis. This completes the proof. □

Hence, we are lead in a natural way to define the new domination parameter

$$\varrho(H) = min\{|P| : P \in \mathcal{P}(H)\},$$

which we call *private clique domination* (*pcd-*) *number* of H; since the logically vacuous case of the implication in (15) means that every independent dominating set of H satisfies (15) and hence must be a $\mathcal{P}$-set, this parameter is well defined for any hypergraph. The following is a straightforward consequence of Theorem 3.1.

Corollary 3.2. *For any strong claw-free hypergraph H, $\varrho(H) \leq \gamma(H)$.*

Problem 3.3. *Characterize hypergraphs satisfying $\varrho(H) = \gamma(H)$.*

Remark 3.4. *It is important to note here that the converse of Corollary 3.2 is not true, whence a hypergraph that might satisfy $\varrho(H) \leq \gamma(H)$ could contain a strong claw. Hence, of special interest would be the problem of characterizing hypergraphs H for which $\varrho(H) = \gamma(H)$.*

Further, if $\mathcal{ID}(H)$ denotes the set of all independent dominating sets of H, each such set being a minimal dominating set of H by virtue of Corollary 1.5, then, $\mathcal{ID}(H) \subseteq \mathcal{D}^m(H)$, and this set inclusion implies

$$\gamma(H) \leq \gamma_i(H) := min\{|D| : D \in \mathcal{ID}(H)\}.$$

We shall now extend the well-known Allan-Laskar theorem for graphs (*cf.*: Haynes et al. [9]) to the realm of hypergraphs.

Theorem 3.5. *If a hypergraph $H = (X, \mathcal{E})$ does not contain any strong $3-$ claw then*

$$\gamma(H) = \gamma_i(H).$$

Proof. For $A \subseteq X$, let $q(A)$ denote the number of non-loop edges contained in the subhyppergraph induced by A, denoted $\langle A \rangle_H$ and defined as the hypergraph $(A, \{E \in \mathcal{E} : E \subseteq A\})$. Let S be any $\gamma(H)$*-set*, that is, a dominating set having cardinality $\gamma(H)$, in H. Without loss of generality, we may assume that $q(S)$ is least possible among all the $\gamma(H)$-sets S in H. If $q(S) = 0$, we are through. So, let $q(S) > 0$ and E be a non-loop edge contained in $\langle S\rangle_H$. Since S is a *minimum* dominating set in H, it must be minimal too. Therefore, for any nonempty proper subset A of S, $S-A$ is not a dominating set of H, by definition. Hence, in particular, $S-E \notin \mathcal{D}(H)$. This implies that for any $c \in X-(S-E)$, we must have $E_1 \in \mathcal{E}_c(H)$ such that $E \cap E_1 \neq \emptyset$ and for every $E' \in \mathcal{E}_c(H)$, one gets $E' \cap (S-E) = \emptyset$. Hence, for any $a \in E \cap E_1$, consider the set $T = (S-\{a\}) \cup \{c\}$. Then, since $q(T) < q(S)$, by the minimality of $q(S)$ it follows that T cannot be a dominating set of H. Therefore, there must exist $d \in X-T$ and $E_2 \in \mathcal{E}_d(H)$ such that $a \in E_2$ and and for any $E' \in \mathcal{E}_d(H)$, $E' \cap (S \cup \{c\}) = \emptyset$. Then, since $E \subseteq S$, we get $E \cap E_1 \cap E_2 = \{a\}$ and for any $b \in E - E_1$, no two vertices of the set $\{b,c,d\}$ are adjacent in H; that is, $\{b,c,d\}$ is a strongly stable (or, equivalently, independent) set in H. Thus, $(E \cup E_1 \cup E_2,\ \{E, E_1, E_2\})$ is a strong 3-claw in H, a contradiction to the hypothesis. This proves the theorem by contraposition. □

Sumner and Moore (*see* Fulman [8]) defined a graph G to be *domination-perfect* if $\gamma(M) = \gamma_i(M)$ for every induced subgraph M of G. This notion is similarly defined for any hypergraph. In general, note that the property of a hypergraph being strongly claw-free is a hereditary property and hence the following consequence of Theorem 3.5 is easy to see.

Corollary 3.6. *Every strong claw-free hypergraph is domination-perfect.*

Problem 3.7. *Characterize domination-perfect hypergraphs.*

Some extensions of the Allan-Laskar theorem are known (*e.g.,* Bollobás and Cockayne [6]; Henning [10]; Acharya and Gupta [3]). This type of extensions of results to hypergraphs would also be worth having towards eventually solving Problem 3.7 and the following problem indicates one instance of such possibilities.

Problem 3.8. *Prove or disprove: If a hypergraph $H = (X, \mathcal{E})$ does not contain a strong $(h+2)$-claw then,*

$$\gamma^h(H) = \gamma_i^h(H),\ h \in \{1, 2, \ldots, r(H)-1\},$$

where $\gamma_i^h(H)$ is the least cardinality of a maximal h-independent set of H and $r(H) = max\{|E| : E \in \mathcal{E}\}$ is the rank *of H.*

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