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Restrained Double Domination Number of a Graph

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Abstract

A set $S \subseteq V(G)$ is a restrained double dominating set for G if every vertex in V is dominated by at least two vertices in S and $\langle V-S \rangle$ has no isolated vertices. The minimum cardinality of a minimal restrained double dominating set is the restrained double domination number and is denoted by $\gamma_{2r}(G)$. In this paper we initiate a study of this parameter and obtain some bounds for $\gamma_{2r}(G)$ and characterize the graphs attaining these bounds. We also derive Nordhaus–Gaddum type results for $\gamma_{2r}(G)$.

Keywords: Domination; Double domination; Restrained domination; Restrained double domination; Restrained double domination number.

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1. Introduction

By a graph $G = (V, E)$ we mean a finite undirected graph without loops or multiple edges. The order and size of G are denoted by p and q respectively. For graph theoretical terms we refer to Harary [4] and for terms related to domination we refer to Haynes et al. [5].

A subset S of V is said to be a dominating set in G if every vertex in $V-S$ is adjacent to at least one vertex in S. The domination number $\gamma(G)$ is the minimum cardinality of a dominating set in G.

The concept of restrained domination was introduced by Telle and Proskurowski [7] indirectly as a vertex partitioning problem. Cyman and Raczek [1] introduced the concept of total restrained domination. A restrained dominating set is a set $S \subseteq V$ where every vertex in $V - S$ is adjacent to a vertex in S as well as to another vertex in $V - S$. The smallest cardinality of a restrained dominating set S of G is called the restrained domination number of G and is denoted by $\gamma_r(G)$. A restrained dominating set S with $|S| = \gamma_r$ is called a γ_r-set.

Harary and Haynes [3] introduced the concept of double domination number of a graph. A set $S \subseteq V$ is a double dominating set for G if every vertex in V is dominated by at least two vertices of S. The minimum cardinality of a double dominating set is the double domination number of G and is denoted by $dd(G)$.

In this paper we define restrained double domination number $\gamma_{2r}(G)$ and initiate a study of this parameter. We obtain some bounds for $\gamma_{2r}(G)$ and characterize the graphs attaining these bounds. We also derive Nordhaus–Gaddum type results for $\gamma_{2r}(G)$. We need the following.

Definition 1.1. *The graph obtained by joining the centres of two stars $K_{1,r}$ and $K_{1,s}$ by an edge is defined to be a bistar and is denoted by $B(r, s)$.*

Theorem 1.2. [2] *Let G be a connected graph of order p. Then $\gamma_r(G) = p$ if and only if G is a star and for any graph G, $\gamma_r(G) = p$ if and only if G is a galaxy (disjoint union of stars).*

Theorem 1.3. [2] *If T is a tree of order $p \geq 3$ then $\gamma_r(T) = p - 2$ if and only if T is obtained from P_4, P_5 or P_6 by adding zero or more pendent vertices to the supports.*

Theorem 1.4. [2] *Let G be a connected graph of order p containing a cycle. Then $\gamma_r(G) = p - 2$ if and only if G is C_4, C_5, or G can be obtained from C_3 by attaching zero or more pendent vertices to at most two of the vertices of the cycle.*

Theorem 1.5. [6] *If G is any connected graph with $\delta(G) = 1$ such that $G \ncong K_{1,p-1}$, then $\gamma_r(G)$ is equal to the number of pendent vertices of G if and only if every nonpendent vertex in G is a support.*

Theorem 1.6. [2] *If $k \geq 1$ is an integer and $r \in \{1, 2, 3\}$ then $\gamma_r(C_{3k+r}) = k + r$.*

2. Main Results

Definition 2.1. *A set $S \subseteq V(G)$ is a restrained double dominating set for G if every vertex in V is dominated by at least two vertices in S and $\langle V - S \rangle$ has no isolated vertices. The minimum cardinality of a minimal restrained double dominating set is called the restrained double domination number of G and is denoted by $\gamma_{2r}(G)$.*

Example 2.2.

(i) $\gamma_{2r}(K_p) = 2$ *for* $p \neq 3$ *and* $\gamma_{2r}(K_3) = 3$.

(ii) $\gamma_{2r}(K_{1,p-1}) = p$.

(iii) *For the Petersen graph* G, $\gamma_{2r}(G) = 6$.

(iv) $\gamma_{2r}(C_p) = p$ *for all* p.

(v) $\gamma_{2r}(W_p) = p - 2k$ *when* $p = 3k + r$, $0 \leq r \leq 2$. *Let* u *be the central vertex of* W_p *and* $v_1, v_2, ...v_{p-1}$ *be the vertices of* C_{p-1}. *By Theorem* 1.6, $\gamma_r(C_p) = k + r$. *Since* u *is adjacent to every other vertex* $\gamma_{2r}(W_p) = \gamma_r(C_p) = k + r = p - 2k$.

(vi) $\gamma_{2r}(K_{m,n}) = 4$ *when* $3 \leq m \leq n$.

Remarks 2.3.

(i) *Every graph* G *without isolated vertices has a restrained double dominating set, as* $V(G)$ *is such a set for* G.

(ii) *If* S *is any restrained double dominating set of* G *then* S *contains all pendent vertices , supports and vertices of degree two. Hence it follows from Theorem 1.5 if* $\gamma_r(G)$ *is equal to the number of pendent vertices then* $\gamma_{2r}(G) = p$.

(iii) *For any graph* G *with* $\delta(G) \geq 3$, $\gamma_{2r}(G) \leq p - 2$.

(iv) *It follows from Theorems 1.3 and 1.4 that there is no graph* G *with* $\gamma_r(G) = \gamma_{2r}(G) = p - 2$.

(v) *For a graph* G *without isolated vertices,* $\gamma_r(G) \leq \gamma_{2r}(G)$ *and by Theorem 1.2,* $\gamma_r(G) = \gamma_{2r}(G) = p$ *if and only if* G *is a galaxy.*

(vi) *If* G *is any graph without isolated vertices and* G *is not a galaxy, then* $\gamma_r(G) \neq \gamma_{2r}(G)$. *By Theorem 1.2, if* $\gamma_{2r}(G) = p$ *then* $\gamma_r(G) < p$. *Suppose* $\gamma_{2r}(G) \leq p - 2$. *Let* S *be a* γ_{2r} *-set of* G *and let* $u \in V - S$. *For any* $w \in N(v) \cap S$, $S - \{w\}$ *is a restrained dominating set of* G *so that* $\gamma_r(G) < \gamma_{2r}(G)$.

Theorem 2.4. *Let* G *be a graph without isolated vertices and let* P *be the set of all pendent vertices and supports of* G *(*P *may be empty). Then* $2 \leq \gamma_{2r}(G) \leq p$. *Lower bound is attained if and only if* $G \cong K_2$ *or* G *has at least two vertices with full degree and* $\delta(G) \geq 3$. *Upper bound is attained if and only if for every edge* (u, v) *in* $\langle V(G) - P\rangle$, *either of* $\deg u$, $\deg v$ *is two or* $\delta(\langle V(G) - \{u, v\}\rangle) = 0$.

Proof. Clearly $2 \leq \gamma_{2r}(G) \leq p$. Suppose $\gamma_{2r}(G) = 2$ and let $S = \{u, v\}$ be a γ_{2r}-set. Then $\deg u = \deg v = p - 1$ and $\langle V - S\rangle$ has no isolated vertices and so $\delta(G) \geq 3$. Converse is obvious.

Assume $\gamma_{2r}(G) = p$. If there exists an edge (u, v) in $\langle V(G) - P\rangle$ with $\deg u \geq 3, \deg v \geq 3$ and $\delta(\langle V(G) - \{u, v\}\rangle) > 0$ then $V(G) - \{u, v\}$ is a restrained double dominating set which is a contradiction.

Conversely let S be a γ_{2r}- set of G .We claim that $S = V$. If not there exists $u, v \in V - S$ such that u and v are adjacent in G. Then $\deg u \geq 3$, $\deg v \geq 3$ and $\delta(\langle V(G) - \{u, v\}\rangle) \neq 0$ which is a contradiction. Hence $\gamma_{2r}(G) = p$. □

We now prove the following theorem whose proof technique is similar to that in [1].

Theorem 2.5. *Let G be a graph without isolated vertices. Then $\gamma_{2r}(G) \geq \frac{5p-2q}{4}$ and the bound is attained for the graph G_1 given in Figure 1.*

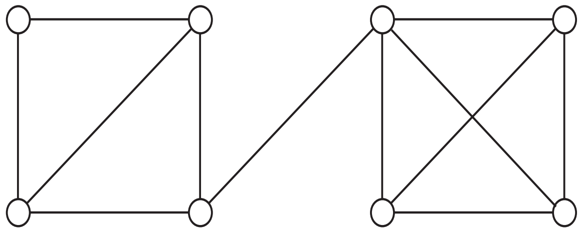

Figure 1:

Proof. Let S be a γ_{2r}-set. Every vertex in $V - S$ is adjacent to at least two vertices in S and one vertex in $V - S$. Also every vertex in S must have at least one neighbor in S. Hence

$$\begin{aligned} q &\geq 2|V-S| + \frac{|V-S|}{2} + \frac{\gamma_{2r}(G)}{2} \\ &= \frac{5}{2}|V-S| + \frac{\gamma_{2r}(G)}{2} \\ &= \frac{5}{2}(p - \gamma_{2r}(G)) + \frac{\gamma_{2r}(G)}{2}. \end{aligned}$$

Thus $2q \geq 5p - 4\gamma_{2r}(G)$, so that $\gamma_{2r}(G) \geq \frac{5p-2q}{4}$. The bound is attained if $G \cong G_1$. □

Theorem 2.6. *If G has no isolated vertices, then $\gamma_{2r}(G) \geq \frac{2p}{\Delta(G)+1}$.*

Proof. Let S be a γ_{2r}-set of G. Let s be the number of edges with one end in S and the other end in $V - S$. Since every vertex in S has at least one neighbor in S,

$$s \leq (\Delta(G) - 1)|S| = (\Delta(G) - 1)\gamma_{2r}(G).$$

Also every vertex in $V - S$ is adjacent to at least two vertices in S and so $s \geq 2|V - S| = 2(p - \gamma_{2r}(G))$.

Thus $2p - 2\gamma_{2r}(G) \leq (\Delta(G) - 1)\gamma_{2r}(G)$ and hence $\gamma_{2r}(G) \geq \frac{2p}{\Delta(G)+1}$.

The bound is attained for the graphs mK_2 and $K_p (p \geq 2)$. So the bound is sharp. □

Theorem 2.7. *Let $G = (X, Y)$ be a connected bipartite graph with $|X| = m$ and $|Y| = n$, $2 \leq m \leq n$. Then $\gamma_{2r}(G) = p$ if and only if either $\Delta(G) \leq 2$ or for every $v \in V(G)$ with $\deg v \geq 3$, whenever $u \in N(v)$ either $\deg u \leq 2$ or u is a support.*

Proof. Let $\gamma_{2r}(G) = p$ and $\Delta(G) \geq 3$. If there exist vertices $u, v \in V(G)$ such that v is not a support, $u \in N(v)$, $\deg v \geq 3$ and $\deg u \geq 3$, then $V(G) - \{u, v\}$ is a restrained double dominating set which is a contradiction.

Converse follows by Remark 2.3 (ii). □

Theorem 2.8. *Let G be a cubic connected graph. Then $\gamma_{2r}(G) = 2$ if and only if $G \cong K_4$ and $\gamma_{2r}(G) = 4$ if and only if $G \cong G_i (1 \leq i \leq 5)$ where G_i are given in Figure 2.*

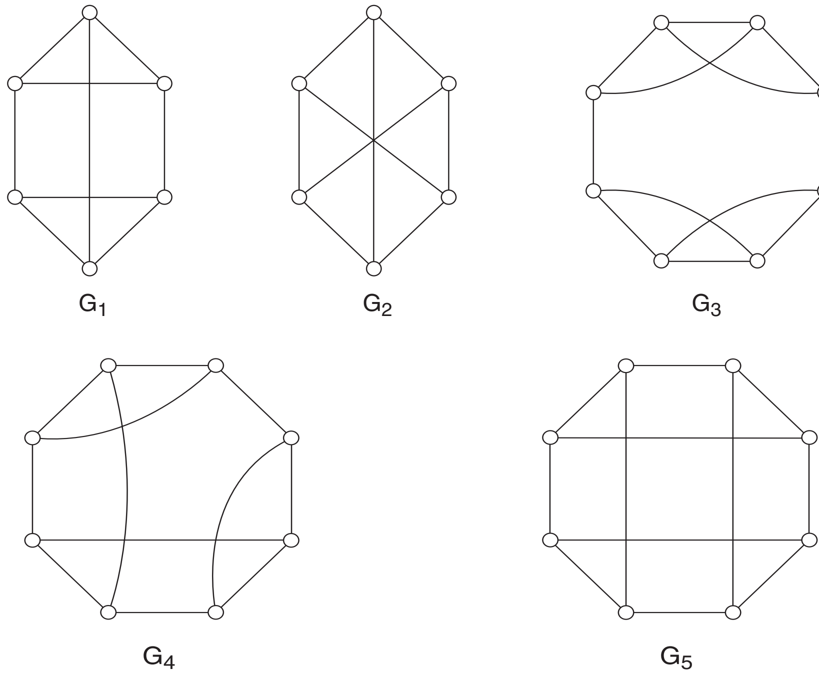

Figure 2:

Proof. Suppose $\gamma_{2r}(G) = 2$. Since G is a cubic graph, by Theorem 2.4, $G \cong K_4$. Converse is obvious.

Suppose $\gamma_{2r}(G) = 4$. Let $S = \{u, v, w, x\}$ be a γ_{2r}-set.

Case (i) $\langle S \rangle$ contains a path of length 3.

In $\langle S \rangle, \deg v = \deg w = 2$. Let y and z be the neighbors of v and w in $V - S$ respectively. Since G is cubic, $|V - S| = 2$. Hence u and x are adjacent. Obviously y and z are adjacent. If y is adjacent to u and z is adjacent to x then $G \cong G_1$. If y is adjacent to x and z is adjacent to u then $G \cong G_2$.

Case (ii) $\langle S \rangle$ does not contain a path of length 3.

In this case $\langle S\rangle \cong 2K_2$. Without loss of generality, let u and v be adjacent and w and x be adjacent. Clearly $|V-S| = 4$. If u and v have two common neighbors in $V-S$ then w and x also have two common neighbors and so $G \cong G_3$.

If u and v have only one common neighbor say s, then w and x have only one common neighbor t.

Let $w_1 \in N(u)\cap(V-S)$ (w_1 is different from t) and $v_1 \in N(v)\cap(V-S)$ (v_1 is different from s). Without loss of generality let $w_1 \in N(w)$ and $v_1 \in N(x)$. If $s \in N(w_1)$ and $v_1 \in N(t)$, then $G \cong G_3$. If $s \in N(v_1)$ and $w_1 \in N(t)$, then also $G \cong G_3$. If $s \in N(t)$ and $w_1 \in N(v_1)$, then $G \cong G_4$.

If u and v do not have common neighbors, then w and x also do not have common neighbors.

Let $u_1 \in N(u)\cap(V-S), v_1 \in N(v)\cap(V-S), w_1 \in N(w)\cap(V-S), x_1 \in N(x)\cap(V-S)$. Without loss of generality let $u_1 \in N(w), v_1 \in N(x), w_1 \in N(u)$ and $x_1 \in N(v)$.

If u_1 and v_1 are adjacent and w_1 and x_1 are adjacent, then $G \cong G_5$. If u_1 and w_1 are adjacent and v_1 and x_1 are adjacent, then $G \cong G_3$. If u_1 and x_1 are adjacent and v_1 and w_1 are adjacent then $G \cong G_5$. Converse is obvious. □

Proposition 2.9. *There exists no connected cubic graph G, with $\gamma_{2r}(G) = 3$.*

Proof. Let S be a γ_{2r}-set with $|S| = 3$. Clearly $\langle S\rangle \cong P_3 = (u, v, w)$. Let y be the neighbor of v in $V-S$. Without loss of generality let $y \in N(u)$. Since y has a neighbor in $V-S$, there exists $x \in V-S$ such that $x \in N(u)\cap N(w)$. But then $\deg w = 2$ and so there exists no connected cubic graph with $\gamma_{2r}(G) = 3$. □

Theorem 2.10. *If T is a tree such that $\bar{T}$ has no isolated vertices, then $\gamma_{2r}(T) + \gamma_{2r}(\bar{T}) \leq 2p$. Equality holds if and only if $T \cong P_4, P_5$ or $B(2,1)$.*

Proof. Obviously, $\gamma_{2r}(T) + \gamma_{2r}(\bar{T}) \leq 2p$. Suppose $\gamma_{2r}(T) + \gamma_{2r}(\bar{T}) = 2p$.

We claim that diam(T) = 3 or 4. Clearly diam(T) ≥ 3. Suppose diam(T) = d $\geq$ 5 and let $v_1, v_2, \ldots, v_{d+1}$ be the diametrical path in T. Now $V(T) - \{v_1, v_{d+1}\}$ is a restrained double dominating set of $\bar{T}$ which is a contradiction and so diam(T) = 3 or 4.

Suppose diam(T) = 3. Let (u, u_1, v_1, v) be the diametrical path. If $\deg u_1 = \deg v_1 = 2, T \cong P_4$. If $\deg u_1 \geq 4$ or if $\deg u_1 = 3$ and $\deg v_1 \geq 3$, then we get a restrained double dominating set of $\bar{T}$ with cardinality $p-2$ and so $T \cong B(2,1)$.

If diam(T) = 4, then $T \cong P_5$, since in all other cases we get a restrained dominating set of $\bar{T}$ with cardinality $p-2$. □

Theorem 2.11. *Let G be a connected unicyclic graph such that $\bar{G}$ has no isolated vertices. Then $\gamma_{2r}(G) + \gamma_{2r}(\bar{G}) \leq 2p$ and equality holds if and only if $G \cong C_4, C_5$ or $G_i (1 \leq i \leq 4)$ where G_i are given in Figure 3.*

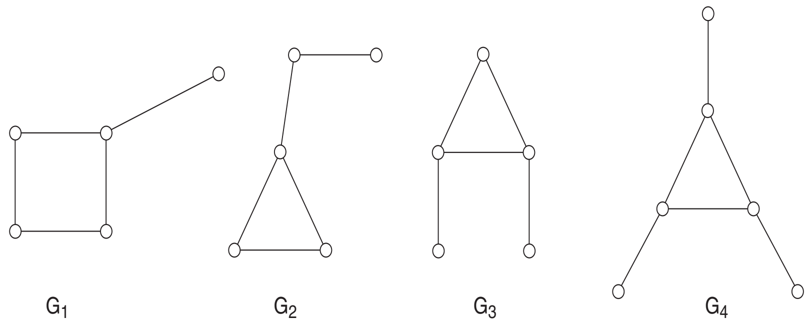

Figure 3:

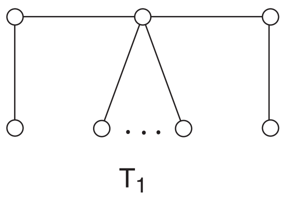

Figure 4:

Proof. Clearly $\gamma_{2r}(G) + \gamma_{2r}(\bar{G}) \leq 2p$. Suppose $\gamma_{2r}(G) + \gamma_{2r}(\bar{G}) = 2p$ and let $C_n = (v_1, v_2, \ldots, v_n = v_1)$ be the cycle in G. If $n \geq 6$, $V(G) - \{v_2, v_5\}$ is a restrained double dominating set of $\bar{G}$ and so $n \leq 5$. If $n = 5$ and if $u \in V(G) - C_n$, then for any $v \in C_n \cap (V - N(u)), V(G) - \{u, v\}$ is a restrained double dominating set of $\bar{G}$ and so $G \cong C_5$. Suppose $n = 4$. If there exists a pendent vertex $u \in V(G) - C_n$ such that $d(u, v_i) \geq 2$ for some $i(1 \leq i \leq 4)$, then $V(G) - \{u, v_{i-1}\}$ is a restrained double dominating set of $\bar{G}$. If G contains two pendent vertices say u and v, then $V(G) - \{u, v\}$ is a restrained double dominiating set of $\bar{G}$. Hence $G \cong C_4$ or G_1. If $n = 3$, as above we can show that every vertex not on C_3 is at most at distance two from a vertex of C_3. If there exists two vertices at distance two from a vertex of C_3, we get a similar contradiction. Also the degree of every vertex of C_3 is either two or three, since otherwise either $\bar{G}$ has isolated vertices or $\bar{G}$ has a restrained double dominating set of cardinality $p - 2$. Hence $G \cong G_2, G_3$ or G_4. Converse is obvious. □

Theorem 2.12. *If T is a tree different from the star $K_{1,p-1}$, then*

$$\gamma_{2r}(\bar{T}) = \begin{cases} 5 & \text{if } T \cong P_5,\ B(r,s) \text{ with } r+s=3 \\ 4 & \text{if } T \cong B(r,s) \text{ with } r+s \neq 3 \text{ or } T_1 \\ & \text{where } T_1 \text{ is given in fig (4).} \\ 3 & \text{otherwise.} \end{cases}$$

Proof. Let S be the set of all supports of T. Since G is different from star, $|S| \geq 2$.

Case (i) $|S| \geq 4$.

Let u_1, v_1, w_1, x_1 be four distinct supports with pendent vertices, $u \in N(u_1), v \in N(v_1), w \in N(w_1)$ and $x \in N(x_1)$. Since T is a tree, at least two of these four supports are nonadjacent. Without loss of generality we can assume that x_1 is nonadjacent to w_1. Let $D = \{u, v, w\}$. Clearly D is a double dominating set in $\bar{T}$ and by choice of x_1, $\langle V - D \rangle$ has no isolated vertices. Hence D is a restrained double dominating set of $\bar{T}$. Also no set of cardinality two can be a restrained double dominating set of $\bar{T}$ and so $\gamma_{2r}(\bar{T}) = 3$.

Case (ii) $|S| = 3$.

Let $S = \{u_1, v_1, w_1\}$ where u, v and w are pendent vertices with $u \in N(u_1)$, $v \in N(v_1)$, $w \in N(w_1)$. Suppose $\langle S \rangle \cong P_3$. If $\deg u_1 \geq 3$ or $\deg w_1 \geq 3$, then $D = \{u, v, w\}$ is a restrained double dominating set in $\bar{T}$. Also no set of cardinality two can be a restrained double dominating set of $\bar{T}$ and so $\gamma_{2r}(\bar{T}) = 3$. If $\deg u_1 = \deg w_1 = 2$ and $\deg v_1 \geq 3$, then $\gamma_{2r}(\bar{T}) = 3$ or T is isomorphic to the tree T_1 given in Figure 4 and $\gamma_{2r}(\bar{T}) = 4$. If $\langle S \rangle \cong K_2 \cup K_1$ or $3K_1$ then there exists a vertex x which is neither a pendent vertex nor a support and x is nonadjacent to at least one of $\{u_1, v_1, w_1\}$. In this case, $D = \{u, v, w\}$ is a γ_{2r}-set in $\bar{T}$ and so $\gamma_{2r}(\bar{T}) = 3$.

Case (iii) $|S| = 2$.

Let $S = \{u_1, v_1\}$ and let u and v be pendent vertices with $u \in N(u_1)$ and $v \in N(v_1)$. If u_1 and v_1 are adjacent then $T \cong B(r, s)$. It can be easily verified that $\gamma_{2r}(\bar{T}) = 5$ if $r + s = 3$ and $\gamma_{2r}(\bar{T}) = 4$ otherwise.

Suppose u_1 and v_1 are nonadjacent. If there exists a vertex x adjacent to both u_1 and v_1, either $T \cong P_5$ in which case $\gamma_{2r}(\bar{T}) = 5$ or $\deg\ u_1(\deg v_1) \geq 3$ and $\{u, u_1, v\}$ $(\{u, v_1, v\})$ is a γ_{2r}-set in $\bar{T}$ so that $\gamma_{2r}(\bar{T}) = 3$. Otherwise $\{u, u_1, v_1\}$ is a γ_{2r}-set and so $\gamma_{2r}(\bar{T}) = 3$. □

Corollary 2.13. *If T is a tree different from the star $K_{1,p-1}$, then $\gamma_{2r}(T) + \gamma_{2r}(\bar{T}) \leq p + 5$ and equality holds if and only if T is P_5, $B(r, s)$, where $r + s = 3$. If $T \ncong P_5$, $B(r, s)$, where $r + s = 3$, then $\gamma_{2r}(T) + \gamma_{2r}(\bar{T}) \leq p + 4$ and equality holds if and only if $T \cong T_1$, T_2, $B(r, s)$, where $r + s \neq 3$. In all other cases, $\gamma_{2r}(T) + \gamma_{2r}(\bar{T}) \leq p + 3$ and this bound is sharp.*

Proof. Follows from Theorem 2.12. Sharpness is exhibited by the graph given in Figure 5. □

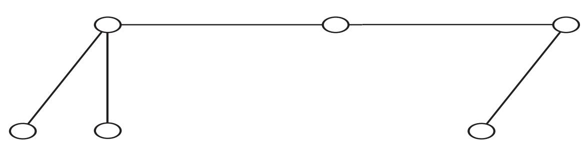

Figure 5:

Theorem 2.14. *Let G be a connected bipartite graph with bipartition X, Y, $|X| = m$, $|Y| = n$ $(2 \leq m \leq n)$ such that $\bar{G}$ has no isolated vertices. If $G \ncong K_{2,3}$ or $K_{3,n}$, then $\gamma_{2r}(\bar{G}) = 3$ or 4. Furthermore $\gamma_{2r}(\bar{G}) = 4$ if and only if every pair of vertices u, v in X, $N(u) \cap N(v) \cap Y \neq \phi$ and vice versa.*

Proof. Clearly $\gamma_{2r}(\bar{G}) \geq 2$. If $\{u, v\}$ is a γ_{2r}-set in $\bar{G}$, then u and v are isolated vertices in G and so $\gamma_{2r}(\bar{G}) \geq 3$. Since $G \ncong K_{2,3}$ or $K_{3,n}$, there exists a set S with $|S \cap X| \geq 2$ and $|S \cap Y| \geq 2$ such that S is a restrained double dominating set of $\bar{G}$ and hence $\gamma_{2r}(G) \leq 4$.

Suppose $\gamma_{2r}(\bar{G}) = 4$. If there exists u, v in X with $N(u) \cap N(v) \cap Y = \phi$, then for any $w \in Y, \{u, v, w\}$ is a γ_{2r}-set in $\bar{G}$, which is a contradiction. Converse is obvious. □

Corollary 2.15. *Let G be a connected bipartite graph such that $\bar{G}$ has no isolated vertices. Then $\gamma_{2r}(G) + \gamma_{2r}(\bar{G}) \leq p + 4$ and the bound is sharp.*

Proof. Follows from Theorem 2.14. Sharpness is exhibited by the graph given in Figure 6. □

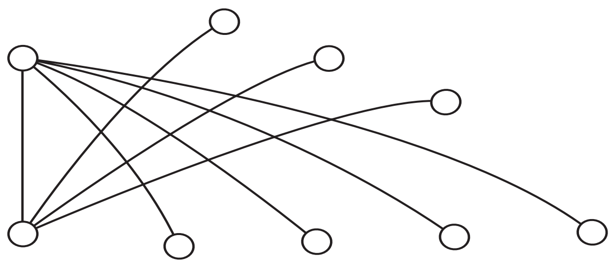

Figure 6:

Acknowledgement

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