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Domination in Hypergraphs: Corrigendum

B.D. Acharya*
Srinivasa Ramanujan Center for Intensification of Interaction in
Interdisciplinary Discrete Mathematics
(SRC-IIIDMS)
University of Mysore
Mysore - 560 005, India.
e-mail: *devadas.acharya@gmail.com*

For standard terminology and notation in hypergraph theory, we refer the reader to C. Berge [2].

A set D of vertices in a hypergraph $H = (X, \mathcal{E})$ is a *dominating set* if for every $x \in X - D$ there exists $y \in D$ such that x and y are *adjacent*, that is, there exists $E \in \mathcal{E}$ such that $x, y \in E$ [1]. The set D is said to be *stable* if it does not contain any edge E of H with $|E| > 1$ and *strongly stable* (or, 'independent') if no two distinct vertices in D are adjacent in H (*see* [2]).

The purpose of this note is to point out some corrections needed in my previous paper [1].

The following theorem is the corrected form of Theorem 1.4 in [1]:

Theorem 1. *Let $H = (X, \mathcal{E})$ be any hypergraph. Then every maximal strongly stable set of H is a minimal dominating set in H and, conversely, every strongly stable dominating set of H is a maximal independent set in H.*

The proof of Theorem 1 goes through the same lines of argument as in the proof of Theorem 1.4 in [1] by replacing the word 'stable' by 'strongly stable' throughout. That Theorem 1.4 in [1] as such does not hold may be reckoned by the counterexample provided by the hypergraph $H = (\{a, b, c, d\}, \{\{a, b, c, d\}\})$ and the maximal stable set $S = \{a, b, c\}$, which is not a minimal dominating set in H. Further, $S' = S - \{c\}$ is a stable dominating set of H that is not even independent.

Next, Corollary 1.13 in [1] is falsified by the simple path $P_5 = (v_1, v_2, v_3, v_4, v_5)$ wherein we have $D = \{v_2, v_4\}$ as a minimal dominating set, which is independent too, whereas we have $N[v_3] \cap D = D$. Therefore, Corollary 1.13 in [1] may safely be ignored and, to maintain the flow of the paper [1], one may then reduce the serial number of every subsequent statement by one in Section 1 of [1]; also, reference to Corollary 1.13 in [1] appearing just after its statement may be read as 'Corollary 1.12' instead.

Lastly, in the proof of Theorem 2.5 in [1], the beginning sentence be amended as follows: "If $t = 1$, this is a well known result for graphs (*c.f.*: Ore [11])". The proof may be continued thereafter by inserting the following

**Address for correspondence*: No.22, 10^{th} Cross, 5^{th} Main, New Thippasandra Post, Malleshpalya, Bangalore-560 075, INDIA.

"However, for hypergraphs in general, the argument may proceed using the following fact, which indeed is a generalization of a well known result in the theory of domination in graphs (*e.g., see* [3], [4]).

Lemma 2. *Let $H = (X, \mathcal{E})$ be any hypergraph and D be any dominating set of H. If D is minimal then, $X - D$ is a dominating set in H.*

Proof. Since D is a minimal dominating set in H, by its characterization Theorem 1.11, for every $d \in D$ there exists $v \in X$ such that

$$N[v] \cap D = \{d\}. \tag{1}$$

If $v \in D$, by (1), we get $v = d$ whence $N[d] \cap (X - D) \neq \emptyset$, for otherwise $N[d] = \{d\}$ which means that d is an isolate in H, contrary to the very definition of a hypergraph. On the other hand, if $v \notin D$ then (1) implies $v \in N[v] \cap (X - D)$. Thus, $N[d] \cap (X - D) \neq \emptyset\ \forall d \in D$, which implies that $X - D$ is a dominating set in H. □

Now, we get back to the case $t = 1$ in the proof of Theorem 2.5. Let D be any minimum dominating set in H (*i.e.*, $|D| = \gamma(H)$). Since D is then a minimal dominating set of H, Lemma 2 implies, $X - D$ is a dominating set in H. Therefore,

$$|D| = \gamma(H) \leq |X - D|$$

$$\Rightarrow \gamma(H) \leq \frac{|X|}{2},$$

as required."

The rest of the proof of Theorem 2.5 in [1] goes through as it is.

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References

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