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A Survey on Radio k-Colorings of Graphs

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Abstract

The frequency assignment problem has been started from the discovery that transmitters, received the same or closely related frequencies, had interferences with one another. Nearly three decade back this problem has been modelled as a graph labelling problem. This labelling has several variations depending upon the type of assignment of frequency to transmitters. Here we shall discuss about some of them and give a detail survey on the most recent frequency assignment problem called the radio k-coloring of graphs.

Keywords: frequency Assignment Problem, Distance Constrained Labelling, Radio k-coloring, Radio k-chromatic Number.

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1. Introduction

In the frequency assignment problem (FAP) the task is to assign radio frequencies to transmitters at different locations without causing interference and also minimizing the span. FAP plays an important role in wireless networking and is a well-studied interesting problem. Due to rapid growth of wireless networks and to the relatively scarce radio spectrum the importance of FAP is growing significantly. Many researchers have modelled FAP as an optimization problem as follows: Given a collection of transmitters to be assigned operating frequencies and a set of interference constraints on transmitter pairs, find an assignment that satisfies all the interference constraints and minimizes the value of a given objective function. In 1980, Hale [18] has modelled FAP as a Graph labelling problem (in particular as a generalized graph coloring problem) and is an active area of research now. This graph theory model of FAP as follows: transmitters are represented by vertices of a graph and those that are very close are joined by edges. The maximum interference occurs among transmitters represented by adjacent vertices. Some interferences still occur among transmitters represented by vertices at distance two, three, etc. Assignment of frequencies to transmitters means assignment of labels to vertices. In order to assign frequencies to transmitters efficiently we have to assign labels to the vertices in such a way that the labels of the adjacent vertices differ a lot, while that of the vertices which are not adjacent differ less and finally we also have to minimize the maximum label assigned. This type of labelling of a graph is also called the *distance-constrained labelling.*

In 1988, Roberts[39] proposed an FAP with two levels of interference which Griggs adapted to graphs and extended to a more general graph problem of distance-constrained labelling [17] as follows. For nonnegative integers $p_1, \ldots, p_m$, an $L(p_1, \ldots, p_m)$-labelling of a graph G is a labelling of its vertices by nonnegative integers such that vertices at distance exactly i receive labels that differ by at least p_i. The maximum label assigned to any vertex is called the *span* of the labelling. The goal of the problem is to construct an $L(p_1, \ldots, p_m)$-labelling of the smallest span. Sometime the distance constraints are considered to decrease with the distance [3], i.e., $p_1 \geq p_2 \geq \cdots \geq p_m \geq 1$. However, there are also practical applications of the case $p_1 = 0,\ p_2 = 1$ [2, 20]. Distance-constrained labelling problem is related to ordinary graph coloring problem of graphs. If $p_1 = \cdots = p_m = 1$, then the problem reduces to the coloring of the k^{th} power of the graph G. So many results on colorings of graph powers translate to distance-constrained labellings and vice versa (for example see [36, 37]).

More popular distance-constrained labelling is the $L(2,1)$-labellings of graphs. A major open problem in this labelling is the conjecture of Griggs and Yeh [17] that asserts that every graph G with maximum degree $\Delta \geq 2$ has an $L(2,1)$-labelling of span at most Δ^2. The conjecture is still open almost 16 years after it was published. Lots of research work have been done on $L(2,1)$-labelling of graphs and their algorithmic aspects (for example see [1, 4, 5, 6, 7, 14, 15, 17, 22, 26, 32, 35]). An extensive survey on $L(2,1)$-labelling of graphs, their variants and on some other coloring problems by Laskar et al. [28] is coming soon. An another version of FAP is described by a graph G in which each edge e is assigned a positive integer weight $w(e)$ and the problem is to give labellings to the vertices of G with positive integrs such that the labels of adjacent vertices u and v differ by at least $w(uv)$, for example, see [27]. A survey on this problem is made by Mc Diarmid [34]. Instead of integer labelling in the above FAP real number labellings are also considered by some researchers, a survey of which can be seen in [16]. By taking a different kind of distance between any two numbers Heuvel et al. [19] have discussed about an FAP and have focussed on the optimal labelling of infinite triangular lattice, infinite square lattice, and infinite line lattice.

In this paper we are interested on the FAP radio k-colorings of graphs, $1 \leq k \leq diam(G)$, suggested by Chartrand et.al [8, 9]. We give a detail survey on this FAP in the next section.

2. Radio k-colorings of Graphs

Motivated by the problem of channel assignment to FM radio stations of Federal Communications Commission of the United States Chartrand et el. [9] have introduced an FAP called radio k-colorings of graphs as defined below. For any positive integer k, $1 \leq k \leq diam(G)$, a *radio k-coloring* f of G is an assignment of positive integers to the vertices of G such that $|f(u) - f(v)| \geq 1 + k - d(u, v)$, for every two distinct vertices u, v of G, where $d(u, v)$ is the distance between u and v in G. The *span* $rc_k(f)$ of a radio k-coloring f of G is the maximum color (positive integer) assigned to a vertex of G. The minimum span of all radio k-colorings of G is called the *radio k-chromatic number* $rc_k(G)$ of G. The radio 1-chromatic number is then the chromatic number $\chi(G)$. If $diam(G) = d$, the radio d-coloring of G is called the *radio coloring* of G, see [8]. Radio d-chromatic number $rc_d(G)$ is called the *radio number* of G and is denoted by simply $rn(G)$. The radio $(d-1)$-coloring of G is known as the *radio antipodal coloring* of G because only antipodal vertices can receive same colors (two vertices u and v in a graph G are said to be antipodal if $d(u, v) = diam(G)$) and the radio $(d-1)$-chromatic number is called the *radio antipodal number* $ac(G)$ of G. The radio $(d-2)$-coloring of G is called the *nearly antipodal coloring* of G and the radio $(d-2)$ chromatic number is called the *nearly antipodal number* $ac'(G)$ of G. Notice that the radio k-coloring of G is a particular case of an $L(p_1, \dots, p_m)$-labelling.

For any simple connected graph G, finding a radio k-coloring of G is not at all difficult. However finding radio k-chromatic number of G is highly non-trivial.

2.1. Known Results on Radio Number of Graphs

In their introductory paper (in 2001), Chartrand et al. [8] have studied radio number of some well known graphs, namely, cycles, complete multipartite graphs, and graphs with diameter 2. They have computed the radio numbers of C_n, for $n \leq 8$, and have given bounds for other values of n. For $n \geq 9$, the lower bound of $rn(C_n)$ is $3\lceil \frac{n}{2} - 1 \rceil$ and the upper bound is $\frac{(n-1)}{2}^2 + 1$ for n odd and $\frac{n}{2}^2 - \frac{n}{2} + 2$ for n even. The radio number of a complete t-partite graph $K_{n_1,\dots,n_t}$ is $\sum_1^t n_i + (t-1)$. It has also been shown by them that if G is a connected graph of order n and diameter 2, then $n \leq rn(G) \leq 2n - 2$ and that for every pair k and n of integers with $n \leq k \leq 2n - 2$, there exists a connected graph of order n and diameter 2 with $rn(G) = k$. A characterization of connected graphs of order n and diameter 2 with prescribed radio number is also presented in [8].

For paths and cycles, the radio number was studied by Chartrand et al. [9] and Zhang [44], while the exact value remained open until solved by Liu and Zhu [33]. They have determined the exact value of radio number of path P_n, $n \geq 4$, as $2p^2 - 2p + 2$ if $n = 2p$ and $2p^2 + 3$ if $n = 2p + 1$. They have also determined the exact value of radio number of C_n, $n \geq 3$, as $\frac{n-2}{2}\phi(n) + 1$ if $n \equiv 0, 2$ $(mod\ 4)$ and $\frac{n-1}{2}\phi(n)$ if $n \equiv 1, 3$ $(mod\ 4)$, where $\phi(n)$ is equal to $s + 1$ if $n = 4s + 1$ and is $s + 2$ if $n = 4s + r$ for some $r = 0, 2, 3$.

Fernandez et al. [13] have given radio number of the complete graph K_n and the wheel graph W_n as n and $n + 2$ respectively, where the wheel graph W_n consists of an n-cycle together with a center vertex that is adjacent to all n vertices of the cycle. They have also computed $rn(G_n) = 4n + 2$, $n \geq 4$, where G_n is the n-gear graph obtained from W_n by inserting one vertex between each pair of vertices on the main cycle of W_n. Equivalently, the n-gear graph consists of a cycle on $2n$ vertices and the center vertex c, with every other vertex on the cycle adjacent to the vertex c.

The square of a graph G is a graph constructed from G by adding edges between vertices of distance two apart in G and is denoted by G^2. The radio number $rn(C_n^2)$, n even, of the square of even cycles are completely determined by Liu and Xie [30]. They are $\frac{2p^2-5p-1}{2}$ if $n = 4p$ and p is odd, $\frac{2p^2+3p}{2}$ if $n = 4p$ and p is even, $p^2 + 5p + 1$ if $n = 4p + 2$ and p is odd and $p^2 + 4p + 1$ if $n = 4p + 2$ and p is even. In case of square of odd cycle C_n^2, $n = 4p + 1$, they have found out that the value of $rn(C_n^2)$ is equal to $p^2 + 2p$ if p is even and is $p^2 + p$ if $p \equiv 3 \pmod 4$. Moreover, for the case $p \equiv 1 (\text{mod } 4)$, they have given bounds as $p^2 + p + 1 \leq rn(C_{4p+1}^2) \leq p^2 + p + 2$. Liu and Xie [31] have determined the exact value of radio number of the square path P_n^2, $n \geq 9$, as $p^2 + 2$ if $n \equiv 1(mod\ 4)$, and $p^2 + 1$ otherwise, where $p = \lfloor \frac{n}{2} \rfloor$.

For a positive integer r the r^{th} power of a graph G, G^r, is the graph on the vertices of G with two vertices u and v adjacent in G^r whenever $d(u, v) \leq r$. Sooryanarayana and Raghunath [40] have determined the radio number of the cube of a cycle, C_n^3, for all $n \leq 20$ and for $n \equiv 0$ or 2 or 4 (mod 6).

Next, Liu [29] has studied the radio number of trees. He has given a lower bound for the radio number $rn(T)$ of an n-vertex tree with diameter d as $(n-1)(d+1)+1-2w(T)$, where $w(T)$ is the weight of T defined as $w(T) = min_{u \in V(T)}\{\Sigma_{v \in V(T)} d(u, v)\}$. He has also characterized the trees achieving this bound. A spider is a tree with at most one vertex of degree more than two. If the spider has no vertex with degree more than two then it is a path and the radio number has already been discussed for this. If the spider has a vertex v of degree more than two, say m, then the spider will have m number of paths with one end at v and the other at a pendant vertex. If the length of these paths be $l_1, l_2, l_3, \ldots, l_m$ with $l_1 \geq l_2 \geq \cdots \geq l_m$, $m \geq 3$, then the spider is denoted by $S_{l_1,l_2,l_3,\ldots,l_m}$. Liu [29] has also given a lower bound for the radio number of $S_{l_1,l_2,l_3,\ldots,l_m}$ as $\displaystyle\sum_{i=1}^{m} l_i(l_1 + l_2 - l_i) + \left\lceil \frac{l_1 - l_2}{2} \right\rceil \left\lfloor \frac{l_1 - l_2}{2} \right\rfloor + 1$ and has characterized the spiders achieving this bound.

2.2. Known Results on Radio k-Chromatic Number of Graphs

As the radio coloring is a particular case of the radio k-coloring, we sometime skip the results for radio k-chromatic number of graph G with $k = diam$(G). Radio k-chromatic number of very few graphs are known so far, even for paths this number has not yet been determined completely.

In [9], Chartrand et al. have introduced the radio k-coloring of graphs and have studied some properties of it. Chartrand et al. [12] have given an upper bound for the radio k-chromatic number, $rc_k(P_n)$, of path P_n on n vertices for $1 \leq k \leq n-1$ as $\frac{k^2+2k+1}{2}$ if k is odd and $\frac{k^2+2k+2}{2}$ if k is even. Also they have given a lower bound for the same as $\frac{k^2+3}{4}$

if k is odd and $\frac{k^2+4}{4}$ if k is even.

Although Chartrand et al. [9] have defined radio k-colorings of graphs G for $1 \leq k \leq diam(G)$, from the mathematical point of view one can also see this problem for $k > diam(G)$ as this may be useful to find radio k-chromatic number of supergraphs H of G with bigger diameter than that of G. Therefore Kchikech et al. [23] have given exact value of $rc_k(P_n)$ for $k \geq n$ as $(n-1)k - \frac{1}{2}n(n-2)+1$ if n is even and $(n-1)k - \frac{1}{2}n(n-1)^2+2$ if n is odd. Also they have improved the lower bound for the same when $1 \leq k \leq n-1$ as $\frac{k^2+3}{2}$ if k is odd and $\frac{k^2+6}{2}$ if k is even. Liu and Zhu [33] determined the exact value of radio $(n-1)$-chromatic number (which is the radio number) of path P_n (that has already been discussed in the previous subsection). Khennoufa and Togni [25] determined the exact value of the radio $(n-2)$-chromatic number (i.e., radio antipodal number $ac(P_n)$) of the path P_n as $2p^2-4p+5$ if $n = 2p$ and $2p^2-2p+3$ if $n = 2p+1$. Kola and Panigrahi [42] have given the exact value of radio $(n-3)$-chromatic number (i.e., radio nearly antipodal number $ac'(P_n)$) of the Path P_n as $2p^2-6p+8$ if $n = 2p$ and $2p^2-4p+6$ if $n = 2p+1$. Also in [43] they have determined the exact value of radio $(n-4)$-chromatic number as $2p^2-6p+9$ if $n = 2p+1$ and have given an improved upper bound for the same as $2p^2-8p+13$ if $n = 2p$. Panigrahi and Kola [38] have improved the upper bound of $rc_k(P_n)$ given in Chartrand et al. [12] as $\frac{k^2+k+2}{2}$ if $\frac{2n-5}{3} \leq k \leq n-3$ and $\frac{k^2+k+4}{2}$ if $\frac{n-4}{2} \leq k < \frac{2n-5}{3}$. Kola and Panigrahi [41] have further improved the upper bound of $rc_k(P_n)$, for $k \geq 5$ and $k+4 \leq n \leq \lfloor \frac{k^2+2k}{2} \rfloor$, as follows. For k odd the upper bound is $\frac{k^2+2s+1}{2}$ when $k+4 \leq n \leq \frac{3k+1}{2}$ and $s = n-k$; $\frac{k^2+k+2}{2}$ when $\frac{3k+1}{2} < n \leq \frac{5k-1}{2}$; and $\frac{k^2+k+2s+4}{2}$ when $\frac{(5+2s)k+1}{2} \leq n \leq \frac{(7+2s)k-1}{2}$, $s = 0, 1, 2, \ldots, \frac{k-5}{2}$. For k even the upper bound is $\frac{k^2+2s+1}{2}$ when $k+4 \leq n \leq \frac{3k}{2}$ and $s = n-k$; $\frac{k^2+k+2}{2}$ when $n = \frac{3k+2}{2}$; $\frac{k^2+k+4}{2}$ when $\frac{3k+4}{2} \leq n \leq \frac{5k+4}{2}$; and $\frac{k^2+k+2s+6}{2}$ when $\frac{(5+2s)k+2s+6}{2} \leq n \leq \frac{(7+2s)k+2s+6}{2}$, $s = 0, 1, 2, \ldots, \frac{k-6}{2}$. In [23] Kchikech et al. have conjectured that the value of $rc_k(P_n)$ is $\frac{k^2+2k+1}{2}$ if k is odd and $\frac{k^2+2k+2}{2}$ if k is even when n goes to infinity. These conjectured values are exactly the upper bounds of $rc_k(P_n)$ given in Chartrand et al. [12]. As Kola and Panigrahi [41] could not improve the upper bound of $rc_k(P_n)$ for $n \geq \frac{k^2+2k+1}{2}$ if k is odd and $\frac{k^2+2k+2}{2}$ if k is even, they feel that the above conjecture may be true.

Let d be the diameter of the cycle C_n on n vertices. We have already mentioned before that Liu and Zhu [33] have determined the exact value of the radio number, $rc_d(C_n)$, of C_n, $n \geq 3$. Juan and Liu [21] have determined $rc_{d-1}(C_n)$ (i.e., the antipodal number $an(C_n)$), of a cycle C_n as $2s^2$ when $n = 4s+1$, $2s^2+2s$ when $n = 4s+3$ and have given an upper and lower bounds when $n = 4s$ as $2s^2-1$ and $2s^2 - \lfloor \frac{s}{2} \rfloor$ respectively. Whereas Chartrand et al. [10] have computed the exact value of the same when $n = 4s+2$. Furthermore, Chartrand et al. [11] have given upper and lower bounds for radio antipodal number of an arbitrary graph in terms of its diameter and other invariants.

Kchikech et al. [24] have given upper and lower bounds for radio k-chromatic number, $rc_k(G \square G')$, of the cartesian product $G \square G'$ of any two graphs G and G' of order n and

m respectively, as below. The upper bound is $m(rc_k(G) + (m-1)k - \sum G' + 1)$ for $k \geq diam(G\Box G') - 1$ and the lower bound is $(mn-1)(k+1) - m\sum_c G - n\sum_c G' + 2$ for $k \geq 1$, where for any graph H with the vertex set $V(H) = \{v_0, \dots v_{p-1}\}$, $\sum H$ and $\sum_c H$ are defined as $\sum H = max_\pi \sum_{i=0}^{p-2} d(\pi(v_{i+1}), \pi(v_i))$, $\sum_c H = max_\pi \sum_{i=0}^{p-1} d(\pi(v_{i+1}), \pi(v_i))$, π is a permutation of $V(H)$ and integers are taken modulo n. Also they have given upper and lower bounds for radio k-chromatic number, $rc_k(Q_n)$, of hypercube Q_n as $(2^n-1)k - 2^{n-1} + 1$ if $n-1 \leq k < 2n-2$ and $(2^n-1)k - 2^{n-1}(2n-3) - n$ if $k \geq 1$.

In [23] Kchikech et al. have found the exact value of the radio k-chromatic number of stars $K_{1,n}$ as $n(k-1)+2$ and have also given an upper bound for radio k-chromatic number, $rc_k(T)$, $k \geq 2$, of an arbitrary tree T on n vertices as $(n-1)(k-1)$.

3. Concluding Remark

The FAP originated in the early 20th century as technology advanced and there are several variations of it expressed as an optimization or a graph labelling problem. Not every type of FAP have been covered in this article. Here we have discussed mainly on the FAP suggested by Chartrand et al. [8, 9]. For early results on FAP one may refer to [18] which not only gives a coverage of several type of FAP but also contains a good number (ninety eight) of bibliographical references. Finding radio k-chromatic number of a graph is highly nontrivial. Improvement of a lower bound is much more difficult than the improvement of an upper bound for a radio k-chromatic number of a graph. To give an upper bound at least we know that we have to define an appropriate radio k-coloring. For improvement of lower bounds some techniques appear in [33] and [23] only. But they are not applicable for any general graph. So finding a technique for improvement of lower bound is a matter of great concern.

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