AKCE J. Graphs. Combin., 6, No. 1 (2009), pp. 31-39

On super d-antimagic labelings of disjoint union of prisms

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Abstract

We label the vertices, edges and faces by the consecutive integers from 1 to $|V|+|E|+|F|$ in such a way that the label of a face and the labels of the vertices and edges surrounding that face all together add up to a weight of that face. These face weights then form an arithmetic progression with common difference d. In this paper we examine the existence of such labeling for a disjoint union of prisms.

Keywords: union of prisms, d-antimagic labeling, super d-antimagic labeling

2000 Mathematics Subject Classification: 05C78

1. Introduction

We consider finite undirected graphs without loops and multiple edges. Denote by $V(G)$ and $E(G)$ the set of vertices and the set of edges of a graph G, respectively. Let $|V(G)| = v$ and $|E(G)| = e$.

If we consider a *plane graph* which is drawn on the Euclidean plane in such a way that edges do not cross each other except at the vertices of the graph then for a plane graph $G = (V, E, F)$, it makes sense to consider its faces, including the unique face of the infinite area. Let $F(G)$ be the face set and let $|F(G)| = f$.

A labeling of a graph is any mapping that sends some set of graph elements to a set of numbers (usually positive integers). Specially, if we have bijections $g : V(G) \cup E(G) \to \{1, 2, \dots, v + e\}$ or $h : V(G) \cup E(G) \cup F(G) \to \{1, 2, \dots, v + e + f\}$ then the labelings are called respectively a *labeling of type* $(1, 1, 0)$ (total labeling) or a *labeling of type* $(1, 1, 1)$.

The *weight* of a face under a labeling is the sum of labels (if present) carried by that face and the edges and vertices on its boundary.

A labeling of plane graph G is called d-*antimagic* if for every number s the set of s-sided face weights is $W_s=\{a_s, a_s+d, a_s+2d, \dots, a_s+(f_s-1)d\}$ for some integers a_s and $d \geq 0$, where f_s is the number of the s-sided faces. We allow different sets W_s for different s.

The d-antimagic labeling of the plane graphs was defined in [2]. In particular for $d = 0$, Ko–Wei Lih in [12] called such labeling *magic* . Ko–Wei Lih [12] described magic labelings of type $(1, 1, 0)$ for the wheels, the friendship graphs and the prisms.

The d-antimagic labelings of type $(1, 1, 1)$ for the generalized Petersen graphs $P(n, 2)$, the hexagonal planar maps and the grids can be found in [4], [5] and [6].

A d-antimagic labeling is called *super* if the smallest possible labels appear on the vertices. The super d-antimagic labelings of type $(1, 1, 1)$, $d \in \{0, 1, 2, 3, 4, 5, 6\}$, for antiprisms are described in [8].

The prism D_n, $n \geq 3$, is a cubic graph which can be represented as a Cartesian product $P_2 \times C_n$ of a path on two vertices with a cycle on n vertices. Prism D_n, $n \geq 3$, consists of an outer n-cycle $y_1, y_2, \dots, y_n$, an inner n-cycle $x_1, x_2, \dots, x_n$, and a set of n spokes x_iy_i, $i = 1, 2, \dots, n$. Note that $x_{n+1} = x_1$ and $y_{n+1} = y_1$. We denote by $z_{4,i}$, $1 \leq i \leq n$, the 4-sided face bounded by the edges x_iy_i, x_ix_{i+1}, $x_{i+1}y_{i+1}$ and y_iy_{i+1}, and we denote by z_n and z'_n the inner n-sided face and outer n-sided face, respectively.

In [2], it was proved that the prism D_n has the d-antimagic labelings of type $(1, 1, 1)$ for $d \in \{2, 3, 4, 6\}$ and $n \equiv 3 \pmod 4$. Lin et al. in [13] showed that D_n, $n \geq 3$, admits d-antimagic labelings of type $(1, 1, 1)$ for $d \in \{2, 4, 5, 6\}$. The d-antimagic labelings of type $(1, 1, 1)$ for prism D_n and for several $d \geq 7$ are described in [16].

In this paper we deal with the super d-antimagic labeling of type $(1, 1, 1)$ for the disjoint union of m copies of prism D_n, denoted by mD_n.

The disconnected graph mD_n, for $n \geq 3$, $m \geq 2$, consists of the vertex set $V(mD_n) = \{x_i^j, y_i^j : 1 \leq i \leq n,\ 1 \leq j \leq m\}$ and the edge set $E(mD_n) = \{x_i^j x_{i+1}^j, y_i^j y_{i+1}^j, x_i^j y_i^j : 1 \leq i \leq n,\ 1 \leq j \leq m\}$, with indices taken modulo n. The face set $F(mD_n)$ contains mn 4-sided faces $\{z_{4,i}^j : 1 \leq i \leq n,\ 1 \leq j \leq m\}$, m inner n-sided faces $\{z_n^j : 1 \leq j \leq m\}$ and one external face. So, $v = 2mn$, $e = 3mn$ and $f = m(n + 1) + 1$.

Suppose that $h : V(mD_n) \cup E(mD_n) \cup F(mD_n) \to \{1, 2, \dots, 6mn + m + 1\}$ is a super d-antimagic labeling of type $(1, 1, 1)$ of the mD_n. The minimum weight of n-sided face is

$$a_n \geq \sum_{i=1}^{n} i + \sum_{i=1}^{n+1} (2mn + i) = (n + 1)(2mn + n + 1).$$

The maximum weight of n-sided face is

$$a_n + (m - 1)d \leq \sum_{i=1}^{n} (2mn - n + i) + \sum_{i=1}^{n+1} (6mn + m - n + i).$$

Thus

$$d \leq \frac{mn(6n+5) - n(2n+1) + m + 1}{m-1}.$$

The upper bound for the possible value of d is very large in this case.

It is easy to see that the minimum possible weight a_4 of the 4-sided face, under the labeling h, is at least $10mn + 25$. On the other hand, the maximum possible weight of a 4-sided face is no more than $38mn + 5m - 11$. Hence,

$$a_4 + (mn-1)d \leq 38mn + 5m - 11$$

and for $n \geq 3$ we get $d < 30$.

It was shown in [1] that graph mD_n admits a super d-antimagic labeling of type $(1,1,1)$ for $d \in \{0,1,2,3,4,5\}$ and for every $m \geq 2$, $n \geq 3$, $n \neq 4$. In this paper we investigate the existence of the super d-antimagic labeling of type $(1,1,1)$ for mD_n and for $d \in \{6,7\}$.

2. Preliminary Results

We will treat the graph mD_n as the composition of three parts: the disjoint union of m outer n-cycles, the disjoint union of m inner n-cycles and the middle part which consists of the 4-sided faces and the spokes. The middle part can be treated as the disjoint union of n-cycles for the labeling purposes. To simplify the proofs, we denote by

C_O: the disjoint union of m outer cycles of all D_n in mD_n, where $V(C_O) = \{y_i^j : 1 \leq i \leq n,\ 1 \leq j \leq m\}$ and $E(C_O) = \{y_i^j y_{i+1}^j : 1 \leq i \leq n,\ 1 \leq j \leq m\}$,

C_I: the disjoint union of m inner cycles of all D_n in mD_n, where $V(C_I) = \{x_i^j : 1 \leq i \leq n,\ 1 \leq j \leq m\}$ and $E(C_I) = \{x_i^j x_{i+1}^j : 1 \leq i \leq n,\ 1 \leq j \leq m\}$,

C_M: the disjoint union of m middle cycles of all D_n in mD_n, where the vertices of the cycles will be the 4-sided faces of mD_n, i.e. $V(C_M) = \{z_{4,i}^j : 1 \leq i \leq n,\ 1 \leq j \leq m\}$ and the edges of the cycles will be the spokes of mD_n, i.e. $E(C_M) = \{z_{4,i}^j z_{4,i+1}^j = x_{i+1}^j y_{i+1}^j : 1 \leq i \leq n,\ 1 \leq j \leq m\}$. Note, that the indices are taken modulo n.

Thus, the labeling denoted by $(\alpha(C_O), \beta(C_I), \gamma(C_M))$ means a combination of the labeling α for the disjoint union of m outer cycles in mD_n, β for the disjoint union of m inner cycles in mD_n and labeling γ for the disjoint union of m middle cycles in mD_n.

To label C_O, C_I and C_M, we will use some known results on (a,d)-edge-antimagic total labeling and an (a,d)-vertex-antimagic total labeling on cycles. Let us recall the definition of these labelings.

Let $G = (V,E)$ be a graph of order v and size e. Under a total labeling g (labeling of type $(1,1,0)$), the associated *edge-weight* is

$$w_g(xy) = g(x) + g(xy) + g(y)$$

for $xy \in E(G)$, and the associated *vertex-weight* is

$$w'_g(x) = g(x) + \sum_{y \in N(x)} g(xy)$$

for $x \in V(G)$, where $N(x)$ denotes the set of neighbors of x.

A total labeling g is called an (a, d)-*edge-antimagic total labeling* of G if the edge-weights form an arithmetic sequence starting from a and having a common difference d, where $a > 0$ and $d \geq 0$ are two fixed integers. A total labeling g with the property that the set of the vertex-weights is $W = \{w'_g(x) : x \in V(G)\} = \{a, a+d, \dots, a+(v-1)d\}$, $a > 0$, $d \geq 0$, is called (a, d)-*vertex-antimagic total labeling*.

The definition of the (a, d)-edge-antimagic total labeling was introduced by Simanjuntak et al. [15] as a natural extension of a notion of the *magic valuation* defined by Kotzig and Rosa in [11]. The definition of the (a, d)-vertex-antimagic total labeling was introduced in [3] as a natural extension of a notion of the *vertex-magic total labeling* defined by MacDougall et al. in [14]. More comprehensive information on edge-antimagic total labelings and vertex-antimagic total labelings can be found in [7] and [10].

We consider mC_n, as a disjoint union of m copies of cycle C_n with the vertex set $V(mC_n) = \{u_i^j : 1 \leq i \leq n,\ 1 \leq j \leq m\}$ and edge set $E(mC_n) = \{u_i^j u_{i+1}^j : 1 \leq i \leq n,\ 1 \leq j \leq m\}$ with the indices taken modulo n.

Dafik et al. [9] showed that the labeling

$$\begin{aligned} g_1(u_i^j) &= (i-1)m + j, && \text{for} \quad 1 \leq i \leq n \quad \text{and} \quad 1 \leq j \leq m \\ g_1(u_i^j u_{i+1}^j) &= (2n - i + 1)m + 1 - j, && \text{for} \quad 1 \leq i \leq n \quad \text{and} \quad 1 \leq j \leq m \end{aligned}$$

is the $(2mn+2, 1)$-edge-antimagic total labeling of mC_n, for every $m \geq 2$ and $n \geq 3$.

It is not difficult to see that, for every $m \geq 2$ and $n \geq 3$, the labeling

$$\begin{aligned} g_2(u_i^j) &= 2g_1(u_i^j) - 1, && \text{for} \quad 1 \leq i \leq n \quad \text{and} \quad 1 \leq j \leq m \\ g_2(u_i^j u_{i+1}^j) &= 2g_1(u_i^j u_{i+1}^j) - 2mn, && \text{for} \quad 1 \leq i \leq n \quad \text{and} \quad 1 \leq j \leq m \end{aligned}$$

is the $(2mn+2, 2)$-edge-antimagic total labeling of mC_n.

Observe that for the 2-regular graphs (and only for the 2-regular graphs) an (a, d)-edge-antimagic total labeling is equivalent to an (a, d)-vertex-antimagic total labeling (for $d = 0$ see [17]). Evidently, for every $m \geq 2$ and $n \geq 3$, the following labeling

$$\begin{aligned} g_3(u_1^j) &= 2g_1(u_n^j u_1^j) - 2mn, && \text{for} \quad i = 1 \quad \text{and} \quad 1 \leq j \leq m \\ g_3(u_i^j) &= 2g_1(u_{i-1}^j u_i^j) - 2mn, && \text{for} \quad 2 \leq i \leq n \quad \text{and} \quad 1 \leq j \leq m \\ g_3(u_i^j u_{i+1}^j) &= 2g_1(u_i^j) - 1, && \text{for} \quad 1 \leq i \leq n \quad \text{and} \quad 1 \leq j \leq m \end{aligned}$$

is the $(2mn+2, 2)$-vertex-antimagic total labeling of mC_n, since all verifications are trivial.

In [9], it is proved that for $m \geq 2$, $n \geq 3$, the graph mC_n admits a $(m(n+1)+3, 3)$-vertex-antimagic total labeling g_4, where

$$
\begin{aligned}
g_4(u_1^j) &= j, && \text{for} \quad i = 1 \quad \text{and} \quad 1 \leq j \leq m\\
g_4(u_i^j) &= m(n-1+i)+j, && \text{for} \quad 2 \leq i \leq n \quad \text{and} \quad 1 \leq j \leq m\\
g_4(u_i^j u_{i+1}^j) &= im + j, && \text{for} \quad 1 \leq i \leq n \quad \text{and} \quad 1 \leq j \leq m.
\end{aligned}
$$

Using the labeling g_2 we define the labeling α_1 for C_O and the labeling β_1 for C_I in the following way

$$
\begin{aligned}
\alpha_1(y_i^j) &= g_2(u_i^j) + 1,\\
\beta_1(x_i^j) &= g_2(u_i^j),\\
\alpha_1(y_i^j y_{i+1}^j) &= g_2(u_i^j u_{i+1}^j) + 4mn,\\
\beta_1(x_i^j x_{i+1}^j) &= g_2(u_i^j u_{i+1}^j) + 4mn - 1,
\end{aligned}
$$

for $1 \leq i \leq n$ and $1 \leq j \leq m$.

Using the labeling g_4, we construct the labeling γ_1 for C_M as follows

$$\gamma_1(x_i^j y_i^j) = g_4(u_i^j u_{i+1}^j) + 2mn, \quad \text{for} \quad 1 \leq i \leq n, \ 1 \leq j \leq m$$

$$
\gamma_1(z_{4,i}^j) = \begin{cases} g_4(u_{i+1}^j) + 2mn, & \text{for} \quad 1 \leq i \leq n-1, \ 1 \leq j \leq m\\ g_4(u_1^j) + 2mn, & \text{for} \quad i = n, \ 1 \leq j \leq m. \end{cases}
$$

It is easy to verify that the labeling α_1, β_1 and γ_1 use each integer from the set $\{1, 2, \ldots, 6mn\}$ exactly once. The following lemma gives a super 7-antimagic labeling of type $(1,1,1)$ of mD_n for a special case, when $n = 4$.

Lemma 2.1. *For every $m \geq 2$, the graph mD_4 admits a super 7-antimagic labeling of type* $(1,1,1)$.

Proof. We consider the labeling $(\alpha_1(C_O), \beta_1(C_I), \gamma_1(C_M))$ for all $m \geq 2$ and $n = 4$. All the weights of the 4-sided faces $z_{4,i}^j$, $1 \leq i \leq 4$, $1 \leq j \leq m$, form an arithmetic progression of the difference 7 with the values $77m+8, 77m+15, \ldots, 105m+1$. On the other hand, all weights of the 4-sided faces z_4^j, $1 \leq j \leq m$, have the same value $96m$.

Now, we swap the vertex label $\beta_1(x_i^j) = 2(i-1)m + 2j - 1$ with the vertex label $\alpha_1(y_i^j) = 2(i-1)m + 2j$, for every $1 \leq i \leq 3$ and $1 \leq j \leq m$, and the edge label $\beta_1(x_3^j x_4^j) = 20m + 1 - 2j$ (respectively, $\beta_1(x_4^j x_1^j) = 18m + 1 - 2j$) with the face label $\gamma_1(z_{4,3}^j) = 15m + j$ (respectively, $\gamma_1(z_{4,4}^j) = 8m + j$), for $1 \leq j \leq m$. The swapping process does not have any impact on the weights of the 4-sided faces $z_{4,i}^j$, however the weights of 4-sided faces z_4^j, $1 \leq j \leq m$, constitute the set $\{81m + 1 + 6j : 1 \leq j \leq m\}$.

If we complete the face labels

$$\sigma_1(z_4^j) = 24m + j, \qquad \text{for} \quad 1 \le j \le m$$
$$\sigma_1(external face) = 25m + 1,$$

then the weights of all 4-sided faces in mD_4 form the arithmetic progression $77m+8, 77m+15, \dots, 105m+1, 105m+8, \dots, 112m+1$. Hence, the resulting labeling is super 7-antimagic of type $(1,1,1)$. □

3. Main Results

In this section we will present d-antimagic labeling of type $(1,1,1)$ of the disjoint union of m copies of prism, for $d \in \{6,7\}$.

Theorem 3.1. *For every $m \ge 2$ and $n \ge 3$, the graph mD_n has a super 7-antimagic labeling of type $(1,1,1)$.*

Proof. For $m \ge 2$ and $n = 4$ the assertion follows from previous Lemma 2.1. Now, we consider the labeling $(\alpha_1(C_O), \beta_1(C_I), \gamma_1(C_M))$ for all $m \ge 2$ and $n \ge 3$, $n \ne 4$. The weights of the 4-sided faces $z_{4,i}^j$, $1 \le i \le n$, $1 \le j \le m$, constitute an arithmetic progression of the difference 7, namely $m(19n+1)+8, m(19n+1)+15, \dots, m(26n+1)+1$ and the common weight for all n-sided faces is equal to $6mn^2$.

If we swap the edge label $\beta_1(x_i^j x_{i+1}^j) = 2m(3n-i+1)+1-2j$ with the face label $\gamma_1(z_{4,i}^j) = m(3n+i)+j$, for $1 \le i \le 2$ and $1 \le j \le m$, then the weights of the 4-sided faces will not be changed, but the weights of the n-sided faces will form the set $\{6mn(n-1)+5m-2+6j : 1 \le j \le m\}$. It suffices to complete the face labeling

$$\sigma_2(z_n^j) = 6mn + j, \qquad \text{for} \quad 1 \le j \le m$$
$$\sigma_2(external face) = m(6n+1)+1,$$

and the resulting labeling is the super 7-antimagic labeling of type $(1,1,1)$. □

Theorem 3.2. *The graph mD_n admits a super 6-antimagic labeling of type $(1,1,1)$ for $2 \le m \le 2n$ and $n \ge 3$, $n \ne 4$.*

Proof. Using the labeling g_3, we construct the labeling γ_2 for the 4-sided faces and spokes of the middle part of the mD_n as follows

$$\gamma_2(x_i^j y_i^j) = 2mn + g_3(u_i^j u_{i+1}^j), \quad \text{for} \quad 1 \le i \le n, \ 1 \le j \le m$$

$$\gamma_2(z_{4,i}^j) = \begin{cases} 2mn + g_3(u_{i+1}^j), & \text{for} \quad 1 \le i \le n-1, \ 1 \le j \le m \\ 2mn + g_3(u_1^j), & \text{for} \quad i = n, \ 1 \le j \le m. \end{cases}$$

Under the labeling $(\alpha_1(C_O), \beta_1(C_I), \gamma_2(C_M))$, the weights of the 4-sided faces $z^j_{4,i}$, $1 \leq i \leq n$, $1 \leq j \leq m$, constitute the set $\{20mn+7, 20mn+13, \dots, 26mn+1\}$ and all the weights of the n-sided faces have the same value $6mn^2$.

If we swap the vertex label $\beta_1(x^j_1) = 2j-1$ with the vertex label $\beta_1(x^{m-j+1}_1) = 2(m-j)+1$ and the face label $\gamma_2(z^j_{4,n}) = 2(n+1)m+2-2j$ (respectively, $\gamma_2(z^j_{4,1}) = 4mn+2-2j$) with the face label $\gamma_2(z^{m-j+1}_{4,n}) = 2mn+2j$ (respectively, $\gamma_2(z^{m-j+1}_{4,1}) = 2m(2n-1)+2j$), for every $1 \leq j \leq m$, then the weights of 4-sided faces will not be changed, but the weights of n-sided faces will constitute an arithmetic progression of the difference 4, namely $\{6mn^2+2m+2-4j : 1 \leq j \leq m\}$.

Only if $m \leq 2n$ then for every k, $1 \leq k \leq m-1$, we swap the edge label $\alpha_1(y^{m-k}_i y^{m-k}_{i+1}) = 6mn - 2(im-k) + 2$ with the edge label $\beta_1(x^{m-k}_i x^{m-k}_{i+1}) = 6mn - 2(im-k) + 1$, for $1 \leq i \leq \lfloor \frac{k+1}{2} \rfloor$, and the vertex label $\alpha_1(y^{m-k}_i) = 2(im-k)$ with the vertex label $\beta_1(x^{m-k}_i) = 2(im-k)-1$, for $2 \leq i \leq \lceil \frac{k+1}{2} \rceil$. Applying the swapping will resulting in changes of the face weights of z^j_n to obtain an arithmetic sequence of the difference 5 for the weights of the n-sided faces.

If we complete the face labels

$$\begin{aligned} \sigma_3(z^j_n) &= m(6n+1)+1-j, \qquad for \quad 1 \leq j \leq m \\ \sigma_3(external face) &= m(6n+1)+1, \end{aligned}$$

then the resulting labeling is a super 6-antimagic labeling of type $(1,1,1)$ as requires. □

4. Conclusion

We have shown that for mD_n there exists the super 7-antimagic labeling of type $(1,1,1)$ for $m \geq 2$, $n \geq 3$, and the super 6-antimagic labeling of type $(1,1,1)$ for $n \geq 3$ $n \neq 4$ and $2 \leq m \leq 2n$. We have tried to find a super 6-antimagic labeling of type $(1,1,1)$ for $n \geq 3$ and for every $m \geq 2$, but so far without success. So, we propose the following

Problem 4.1. *For the graph mD_n determine if there is a super 6-antimagic labeling of type $(1,1,1)$, for every $m \geq 2$ and $n \geq 3$.*

In Introduction we have shown that the upper bound for the feasible values of the difference d is 29. Let us recall the open problem from the paper [1].

Problem 4.2. *Find other possible values of the difference d and the corresponding super d-antimagic labelings of type $(1,1,1)$ for mD_n, $m \geq 2$, $n \geq 3$ and $d > 7$.*

Acknowledgement

Support of Slovak VEGA Grant 1/4005/07 and Higher Education Commission Pakistan Grant HEC(FD)/2007/555 is acknowledged.

References

[1] G. Ali, M. Bača, F. Bashir and A. Semaničová-Feňovčíková, On face antimagic labelings of disjoint union of prisms, *Utilitas Math.*, (To appear).

[2] M. Bača and M. Miller, On d-antimagic labelings of type $(1, 1, 1)$ for prisms, *J. Combin. Math. Combin. Comput.*, **44** (2003), 199–207.

[3] M. Bača, F. Bertault, J.A. MacDougall, M. Miller, R. Simanjuntak and Slamin, Vertex-antimagic total labelings of graphs, *Discuss. Math. Graph Theory,* **23** (2003), 67–83.

[4] M. Bača, S. Jendroľ, M. Miller and J. Ryan, Antimagic labelings of generalized Petersen graphs that are plane, *Ars Combin.,* **73** (2004), 115–128.

[5] M. Bača, E.T. Baskoro, S. Jendroľ and M. Miller, Antimagic labelings of hexagonal planar maps, *Utilitas Math.*, **66** (2004), 231–238.

[6] M. Bača, Y. Lin and M. Miller, Antimagic labelings of grids, *Utilitas Math.,* **72** (2007), 65–75.

[7] M. Bača and M. Miller, *Super Edge-Antimagic Graphs: A Wealth of Problems and Some Solutions*, BrownWalker Press, Boca Raton, Florida, 2008.

[8] M. Bača, F. Bashir and A. Semaničová, Face antimagic labelings of antiprisms, *Utilitas Math.,* (To appear).

[9] Dafik, M. Miller, J. Ryan and M. Bača, On super (a, d)-edge antimagic total labeling of disconnected graphs, *Discrete Math.,* (To appear).

[10] J. Gallian, A dynamic survey of graph labeling, *The Electronic Journal of Combinatorics*, **14** (2007), #DS6.

[11] A. Kotzig and A. Rosa, Magic valuations of finite graphs, *Canad. Math. Bull.,* **13** (1970), 451–461.

[12] Ko–Wei Lih, On magic and consecutive labelings of plane graphs, *Utilitas Math.,* **24** (1983), 165–197.

[13] Y. Lin, Slamin, M. Bača and M. Miller, On d-antimagic labelings of prisms, *Ars Combin.,* **72** (2004), 65–76.

[14] J.A. MacDougall, M. Miller, Slamin and W. Wallis, Vertex-magic total labelings of graphs, *Utilitas Math.,* **61** (2002), 68–76.

[15] R. Simanjuntak, F. Bertault and M. Miller, Two new (a, d)-antimagic graph labelings. *Proc. of Eleventh Australasian Workshop on Combinatorial Algorithms* (2000), 179–189.

[16] K.A. Sugeng, M. Miller, Y. Lin and M. Bača, Face antimagic labelings of prisms, *Utilitas Math.,* **71** (2006), 269–286.

[17] W.D. Wallis, *Magic Graphs*, Birkhäuser, Boston - Basel - Berlin, 2001.