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# Construction characterizations for defect $n$-extendable bipartite graphs*

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**Abstract**

A near perfect matching is a matching covering all but one vertex in a graph. Let $G$ be a connected graph and $n \leq (|V(G)|-2)/2$ be a positive integer. If any $n$ independent edges in $G$ are contained in a near perfect matching, then $G$ is said to be defect $n$-extendable. This paper presents two construction characterizations of defect $n$-extendable bipartite graphs and a necessary condition for minimal defect $n$-extendable bipartite graphs.

**Keywords:** Near perfect matching, defect $n$-extendable, minimal defect $n$-extendable graph.

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## 1. Terminology and Introduction

All graphs considered in this paper are undirected, finite and simple. A perfect matching is a matching covering all vertices in a graph. A near perfect matching is a matching

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covering all but one vertex in a graph. Let $G$ be a connected graph and $n \leq (|V(G)|-2)/2$ be a positive integer. If any $n$ independent edges in $G$ are contained in a perfect matching, then $G$ is *$n$-extendable*. If any $n$ independent edges in $G$ are contained in a near perfect matching, then $G$ is *defect $n$-extendable*. A defect $n$-extendable graph $G$ is minimal if for any $e \in E(G)$, $G-e$ is not defect $n$-extendable. An edge $e$ in a defect $n$-extendable graph $G$ is said to be *$n$-deletable* if $G-e$ is still defect $n$-extendable.

We use $G$=($U$,$W$) to denote a bipartite graph $G$ with bipartitions $U$ and $W$. A bipartite graph $G$=($A$,$B$) is said to have *positive surplus* (as viewed from $A$) if the number of neighbors of $X$ is bigger than the size of $X$ for any non-empty subset $X$ of $A$. If the number of neighbors of $X$ is not less than the size of $X$ for any non-empty subset $X$ of $A$, then $G$=($A$,$B$) is said to have *non-negative surplus* (as viewed from $A$). Let $G$ be a graph. Then $c(G)$ and $\nu(G)$ denote the number of components in $G$ and the matching number of $G$ respectively. Some notations appeared in Gallai-Edmonds decomposition of graphs [5] in terms of maximum matching are recalled as follows.

For any graph $G$, let $D(G)$ denote the set of vertices in $G$ that are not covered by at least one maximum matching, $A(G)$ the set of vertices in $V(G)\backslash D(G)$ adjacent to at least one vertex in $D(G)$ and $C(G) = V(G)\backslash(D(G) \cup A(G))$.

For the other terminology and notations not defined in this paper, the reader is referred to [1].

The concept of $n$-extendable graphs was introduced by Plummer [6] and a considerable amount of researches have been done in this area until now (see [7,8]). In order to naturally extend the property of $n$-extendability to graphs of odd order, Lou and Wen [9] introduced the concept of defect $n$-extendable graph. They showed that the connectivity of a defect $n$-extendable graph can be any integer. While Plummer [6] proved that the connectivity of a $n$-extendable graph is not less than $n$+1, which implies that the results on defect $n$-extendable graphs may not be deduced trivially from those of $n$-extendable graphs.

In fact, a few results on defect $n$-extendable graphs have been established until now.

In [3], Little, Grant and Holton gave two characterizations of defect 1-extendable graphs which were called 1-covered graph in their paper. To combine the concept of $n$-extendable graphs and $k$-critical graphs, Liu and Yu [4] introduced ($k$,$n$,$d$)-graphs such that (0,$n$,1)-graphs are the same as defect $n$-extendable graphs. They gave a Tutte style characterization and a property of ($k$,$n$,$d$)-graphs which can directly deduce a characterization of defect $n$-extendable graphs. Lou and Wen found the path decomposition of defect 1-extendable bipartite graphs in [10] and presented the characterization of defect $n$-extendable bipartite graphs with different connectivities [9].

In this paper, we present two construction characterizations for defect $n$-extendable bipartite graphs and a sufficient condition for minimal defect $n$-extendable bipartite graphs.

## 2. Preliminary Results

In this section, we introduce some results which will be used in proof of the main results in this paper.

**Lemma 1.1.** (Wen and Lou [9]) *Let $n$ be a positive integer and $G = (U, W)$ be a defect $n$-extendable bipartite graph with $|W| = |U| + 1$. Then for any $w \in W$, each component in $G - w$ is $k$-extendable where $k = min(\kappa(G) - 1, n - 1)$.*

**Lemma 1.2.** (Hall [2]) *Let $G$=$(X, Y)$ be a bipartite graph. Then $G$ has a matching of $X$ into $Y$ if and only if $|\Gamma_G(S)| \geq |S|$ for all $S \subseteq X$.*

**Lemma 1.3.** (The Gallai-Edmonds structure theorem [5]) *If $G$ is a graph, then $\nu(G) = 1/2(|V(G)| - c(D(G)) + |A(G)|)$.*

**Lemma 1.4.** (The Gallai-Edmonds structure theorem of bipartite graphs [5]) *Let $G = (U_1, U_2)$ be a bipartite graph and for $i = 1, 2$, $A_i = A(G) \cap U_i$ and $D_i = D(G) \cap U_i$. Then the subgraph induced by $A_1 \cup D_2$ and $A_2 \cup D_1$ have positive surplus when viewed from $A_1$ and $A_2$ respectively.*

## 3. Main Results

In this section, we give a recursive construction method for defect $n$-extendable bipartite graphs. Firstly, we introduce the concept of defect $n$-transversal pair.

Let $n$ be a positive integer and $H$=$(U_1$,$U_2)$ be a graph such that each matching of size $n$ is contained in a near perfect matching of $H$ and $|U_2| = |U_1|$+1. A pair of vertex set $(S_1$,$S_2)$ ($S_i \subseteq U_i$, $i$=1,2) is called a *defect $n$-transversal pair* of $H$ if the following statements hold:

1. For any matching $M$ of size $n$+1, $D(H - V(M)) \cap S_i \neq \phi$, $i$=1,2, whenever $D(H - V(M)) \cap U_1 \neq \phi$.
2. For any matching $M$ of size $n$ and any vertex $s_1 \in S_1$ that are not covered by $M$, $D(H - V(M) - s_1) \cap S_2 \neq \phi$.
3. For any matching $M$ of size $n$ and any vertex $s_2 \in S_2$ that are not covered by $M$, we have $D(H - V(M) - s_2) \cap S_1 \neq \phi$ whenever $D(H - V(M) - s_2) \neq \phi$.

**Theorem 1.5.** *A bipartite graph $G = (U_1, U_2)$ is a defect $(n+1)$-extendable graph ($n \geq 1$), then for any edge $e$=$xy$ ($x \in U_1$ and $y \in U_2$) of $G$, $(\Gamma_G(y) \backslash \{x\}$, $\Gamma_G(x) \backslash \{y\})$ is a defect $n$-transversal pair of $G - x - y$.*

*Proof.* Let $G$=($U_1$,$U_2$) be a defect ($n$+1)-extendable graph ($n$ ≥1), $e$=$xy$ ($x \in U_1$ and $y \in U_2$) be an edge in $G$ and $H = G - x - y$. It's obvious that any matching of size $n$ in $H$ is contained in a near perfect matching of $H$. Let $S_1$=$\Gamma_G(y)\backslash\{x\}$ and $S_2$=$\Gamma_G(x)\backslash\{y\}$. It suffices to prove that ($S_1$,$S_2$) is a defect $n$-transversal pair of $H$.

(1) Select any matching $M$ of size ($n$+1) in $H$. Assume $D(H - V(M))\cap U_1 \neq \phi$. Since $G$ is defect ($n$+1)-extendable, there is a near perfect matching $M'$ in $G$ containing $M$. Let $w$ be the $M'$-unsaturated vertex of $G$.

Suppose $xy \in M'$. Then $M'\backslash\{xy\}$ is a near perfect matching in $H$ containing $M$ and hence $D(H - V(M))\cap U_1 = \phi$, a contradiction to $D(H - V(M))\cap U_1 \neq \phi$.

So there is a vertex $s_2 \in U_2$ such that $s_2 \neq y$ and $xs_2 \in M'$. Suppose $y = w$. Then $M'\backslash\{xs_2\}$ is a near perfect matching in $H$ containing $M$ and hence $D(H-V(M))\cap U_1 = \phi$, a contradiction to $D(H - V(M))\cap U_1 \neq \phi$.

So $y \neq w$. Then there is a vertex $s_1 \in U_1$ such that $s_1 \neq x$ and $ys_1 \in M'$. Note that $M''$=$M'\backslash(\{xs_2,\ ys_1\}\cup M)$ is a matching of $H - V(M)$ and $s_1$ is the only one $M''$-unsaturated vertex of $H - V(M)$ in $U_1$. Further, $D(H - V(M))\cap U_1 \neq \phi$ implies $M''$ is a maximum matching of $H - V(M)$. Clearly, $s_i \in S_i$ and $s_i$ are the $M''$-unsaturated vertices in $H - V(M)$, i.e., $s_i \in D(H - V(M))$, $i$=1,2. So $s_i \in D(H - V(M))\cap S_i$ and hence $D(H - V(M))\cap S_i \neq \phi$, $i$=1,2.

(2) Choose any matching $M$ of size $n$ in $H$ and any vertex $s_1 \in S_1$ that is not covered by $M$. Since $G$ is defect ($n$+1)-extendable, there is a near perfect matching $M'$ in $G$ such that $M\cup\{ys_1\}\subseteq M'$. It's obvious that there is an edge $xs_2 \in M'$ where $s_2 \in S_2$. Note that $H - V(M) - s_1 = G - x - y - s_1 - V(M)$ has a maximum matching $M''$=$M'\backslash(M\cup\{ys_1,xs_2\})$, which does not cover $s_2$. Then $s_2 \in D(H - V(M) - s_1)$, i.e., $D(H - V(M) - s_1) \cap S_2 \neq \phi$.

(3) Choose any matching $M$ of size $n$ in $H$ and any vertex $s_2 \in S_2$ that is not covered by $M$. Since $G$ is defect ($n$+1)-extendable, there is a near perfect matching $M'$ such that $M\cup\{xs_2\}\subseteq M'$.

Assume $D(H - V(M) - s_2) \neq \phi$. Let $v$ be the $M'$-unsaturated vertex in $G$. Suppose $y = v$. Then $M'\backslash(M\cup\ \{\ xs_2\ \})$ is a perfect matching of $H - V(M) - s_2$ and hence $D(H - V(M) - s_2) = \phi$, a contradiction to the assumption $D(H - V(M) - s_2) \neq \phi$. So $y \neq v$ and hence there is an edge $ys_1 \in M'$ where $s_1 \in S_1$.

It's not difficult to see that $M''$=$M' - M - \{ys_1,xs_2\ \}$ is a matching in $H - V(M) - s_2$ with only two $M''$-unsaturated vertices. So by $D(H - V(M) - s_2) \neq \phi$, we have $M''$ is a maximum matching of $H - V(M) - s_2$. Note that $M''$ doesn't cover $s_1$. So $s_1 \in D(H - V(M) - s_2)$, i.e., $D(H - V(M) - s_2) \cap S_1 \neq \phi$. □

**Theorem 1.6.** *Let* $G$=($U_1$,$U_2$) *be a connected bipartite graph with* $|U_2| = |U_1| + 1$. *If there exists an edge* $e$=$xy$($x \in U_1$ *and* $y \in U_2$) *of* $G$ *such that each matching of size* $n$ *in* $H = G - x - y$ *is contained in a near perfect matching of* $H$ *and* ($\Gamma_G(y)\backslash\{x\}$, $\Gamma_G(x)\backslash\{y\}$) *is a defect* $n$*-transversal pair of* $H$, *then* $G$ *is defect* $(n + 1)$*-extendable.*

*Proof.* As defined in the theorem, the bipartite graph $G$ is obtained from $H$ by adding a pair of new vertices $x$ and $y$ and a new edge $xy$, and connecting $x$ to all vertices in $S_2$=$\Gamma_G(x)\backslash\{y\}$ and $y$ to all vertices in $S_1$=$\Gamma_G(y)\backslash\{x\}$. Choose any matching $M$ of size $n$+1 in $G$. It suffices to show that there is a near perfect matching in $G$ containing $M$. There are two cases to be considered:

**Case 1.** $M \subseteq E(H)$.

Let $H'$=$H-V(M)$. If $H'$ has a near perfect matching $M'$, then $\{xy\}\cup M\cup M'$ is a near perfect matching in $G$ containing $M$.

If $H'$ has no near perfect matching, then since any $n$ matching of $H$ is contained in a near perfect matching of $H$ and by Lemma 1.3, we have

$$1< c(D(H'))-|A(H')| = |V(H')|-2\nu(H') \leq 3$$

By the parity, the above equality holds. Let $A_i = A(H') \cap U_i$ and $D_i = D(H')\cap U_i$, $i$=1,2. Denote ($A_1$,$D_2$) and ($A_2$,$D_1$) the subgraph induced by the $A_1 \cup D_2$ and $A_2 \cup D_1$ respectively. By Lemma 1.4, both ($A_1$,$D_2$) and ($A_2$,$D_1$) have positive surplus (as viewed from $A_i$). So it's not difficult to see that $|D_2| - |A_1|$=2 and $|D_1| - |A_2|$=1. So $D_1 \neq \phi$ and hence $D(H')\cap U_1 = D_1 \neq \phi$. Further, since ($S_1$,$S_2$) is a defect $n$-transversal pair of $H$, we have $D(H')\cap S_i \neq \phi$, i.e, $D_i \cap S_i \neq \phi$, $i$=1,2. Choose any vertex $s_i \in D_i \cap S_i$. So ($A_2$,$D_1\backslash\{s_1\}$) and ($A_1$,$D_2\backslash\{s_2\}$) have non-negative surplus. Thus by Lemma 1.2, ($A_2$,$D_1\backslash\{s_1\}$) has a perfect matching $M_1$ and ($A_1$,$D_2\backslash\{s_2\}$) has a near perfect matching $M_2$. Clearly, $M_1 \cup M_2 \cup M\cup\{xs_2,ys_1\}$ is a near perfect matching in $G$ containing $M$.

**Case 2.**$M \not\subseteq E(H)$.

Obviously, $M$ contains a matching $M'$ of size $n$ such that $M'\subseteq E(H)$. If $xy\in M$, it's immediate that $M$ is contained in a near perfect matching of $G$. If $xy\notin M$, then $xs_2 \in M$ or $ys_1 \in M$ holds where $s_i \in S_i$.

Suppose $xs_2 \in M$. Assume there is a perfect matching $M''$ in $H - V(M')-s_2$, then $M'\cup M'' \cup\{xs_2\}$ is a near perfect matching in $G$ containing $M$.

Assume there is no perfect matching in $H - V(M')-s_2$. Then $D(H - V(M')-s_2) \neq \phi$ and hence by ($S_1$,$S_2$) being a defect $n$-transversal pair of $H$, we have $D(H-V(M')-s_2)\cap S_1 \neq \phi$. Further, note that each matching of size $n$ in $H$ is contained in a near perfect matching of $H$, so only two vertices in $H - V(M')-s_2$ are not covered by the maximum matching of $H - V(M')-s_2$, one is in $V(H) \cap U_1$, the other is in $V(H) \cap U_2$. Let $A_i = A(H-V(M')-s_2)\cap U_i$ and $D_i = D(H-V(M')-s_2)\cap U_i$, $i$=1,2. By Lemma 1.4, ($A_2$,$D_1$) and ($A_1$, $D_2$) has positive surplus (as viewed from $A_i$). So $|D_1|-|A_2|$=1 and $|D_2|-|A_1|$=1. For any $w \in D_1 \cap S_1$, by Lemma 1.2, we have that ($A_2$, $D_1\backslash$ $\{w\}$) has a perfect matching $M_1$ and $(A_1, D_2)$ has a near perfect matching $M_2$. So $M\cup\{yw\}\cup M_1 \cup M_2$ is a near perfect matching in $G$ containing $M$.

Suppose $ys_1 \in M$. Note that each matching of size $n$ in $H$ is contained in a near perfect matching of $H$, so only two vertices in $H - V(M')-s_1$ are not covered by the maximum matching of $H-V(M')-s_1$ and both of them are in $V(H)\cap U_2$. Let $A_i = A(H-$

$V(M')-s_1)\cap U_i$ and $D_i = D(H-V(M')-s_1)\cap U_i$, $i$=1,2. By Lemma 1.4, $(A_2,D_1)$ and $(A_1, D_2)$ have positive surplus (as viewed from $A_i$). So $|D_1| = |A_2|$=0 and $|D_2| - |A_1|$=2. For any $v \in D_2 \cap S_2$, by Lemma 1.2, we have that $(A_1, D_2\backslash \{v\})$ has a near perfect matching $M_3$. So $M\cup\{xv\}\cup M_3$ is a near perfect matching in $G$ containing $M$. □

By Theorem 1.5 and 1.6, it's not difficult to get the following corollary, which in fact is a recursive method to construct a defect $n$-extendable bipartite graph.

**Corollary 1.7.** *Let $G$=$(U_1$,$U_2)$ be a connected bipartite graph with $|U_2| = |U_1| + 1$. $G$ is defect $(n + 1)$-extendable if and only if there exists an edge $e$=$xy$ ($x \in U_1$ and $y \in U_2$) of $G$ such that each matching of size $n$ in $H = G - x - y$ is contained in a near perfect matching of $H$ and $(\Gamma_G(y)\backslash\{x\}$, $\Gamma_G(x)\backslash\{y\})$ is a defect $n$-transversal pair of $H$.*

Let $(S_1$,$S_2)$ be a defect $n$-transversal pair of $H$. If for any $s_1 \in S_1$ and $s_2 \in S_2$, $(S_1\backslash\{s_1\}$,$S_2)$ and $(S_1$,$S_2\backslash\{s_2\})$ are not defect $n$-transversal pair of $H$, then $(S_1$,$S_2)$ is said to be the minimal defect $n$-transversal pair of $H$. The following theorem shows the relation between minimal defect $(n$+1)-extendable bipartite graphs and minimal defect $n$-transversal pair.

**Theorem 1.8.** *If a bipartite graph $G = (U_1, U_2)$ is minimal defect $(n + 1)$-extendable, then for any edge $xy(x \in U_1$ and $y \in U_2)$ of $G$, we have each matching of size $n$ in $H = G - x - y$ is contained in a near perfect matching of $H$ and $(\Gamma_G(y)\backslash\{x\}$, $\Gamma_G(x)\backslash\{y\})$ is a minimal defect $n$-transversal pair of $H$; and for every $n$-deletable edge $e$ of $H$, $(\Gamma_G(y)\backslash\{x\}$, $\Gamma_G(x)\backslash\{y\})$ is not a defect $n$-transversal pair of $H - e$.*

*Proof.* Let bipartite $G$=$(U_1$,$U_2)$ be minimal defect $(n$+1)-extendable, $xy$ be an edge in $E(G)$ where $x \in U_1$ and $y \in U_2$ and $H = G - x - y$. It's obvious that each matching of size $n$ in $H$ is contained in a near perfect matching of $H$. Let $S_1$=$\Gamma_G(y)\backslash\{x\}$ and $S_2$=$\Gamma_G(x)\backslash\{y\}$. By Theorem 1.5, $(S_1$,$S_2)$ is a defect $n$-transversal pair of $H$. Now we prove $(S_1, S_2)$ is minimal. Suppose, to the contrary, $(S_1$,$S_2)$ is not minimal. Then there is a vertex $u \in S_1$ or $v \in S_2$ such that $(S_1\backslash\{u\}$,$S_2)$ or $(S_1$,$S_2\backslash\{v\})$ is a defect $n$-transversal pair of $H$. If $(S_1\backslash\{u\}$,$S_2)$ is a defect $n$-transversal pair of $H$. Then $e$=$yu\in E(G)$ and hence by Theorem 1.6, $G - e$ is defect $(n$+1)-extendable, contradicting that $G$ is minimal defect $(n$+1)-extendable. Similarly, if $(S_1$,$S_2\backslash\{v\})$ is a defect $n$-transversal pair of $H$, then $e'$=$xv\in E(G)$ and $G - e'$ is defect $(n$+1)-extendable, contradicting that $G$ is minimal defect $(n$+1)-extendable. So $(S_1, S_2)$ is minimal defect $n$-transversal pair of $H$.

Let $e$ be an $n$-deletable edge in $H$. Now we prove that $(S_1$,$S_2)$ is not a defect $n$-transversal pair of $H - e$. Suppose, to the contrary, $(S_1$,$S_2)$ is a defect $n$-transversal pair of $H - e$. Then by Theorem 1.6, $G - e$ is defect $(n$+1)-extendable, contradicting that $G$ is minimal defect $(n$+1)-extendable. □

**Theorem 1.9.** *Let $G = (U, W)$ be a bipartite graph such that $|W| = |U|+1$ and $\kappa(G) \geq n$. Construct a graph $G'$ from $G$ by adding a vertex $x \notin V(G)$ and joining $x$ to all the vertices in a set $S$ which satisfies $S \subseteq W$ and for any matching $M$ of size $n$ in $G$, $S \cap D(G - V(M))\neq \phi$. Then $G$ is defect $n$-extendable if and only if $G'$ is $n$-extendable.*

*Proof.* First, we prove the necessity. Assume $G$ is defect $n$-extendable. Select any matching $M$ of size $n$ in $G'$. It suffices to prove that there is a perfect matching in $G'$ containing $M$. We discuss two cases as follows.

**Case 1.** $x \in V(M)$.

Assume $xy \in M$. Then $M' = M \backslash \{xy\} \subseteq E(G)$ and $|M'| = n-1$. Since $G$ is defect $n$-extendable with $\kappa(G) \geq n$, so by Lemma 1.1, we have $G - y$ is $(n-1)$-extendable. So there is a perfect matching $M''$ in $G - y$ containing $M'$. Clearly $M'' \cup \{xy\}$ is a perfect matching in $G'$ containing $M$.

**Case 2.** $x \notin \mathrm{V}(M)$.

Then $M \subseteq E(G)$. Since $G$ is defect $n$-extendable and $\Gamma_{G'}(x) \cap D(G - V(M)) \neq \phi$, then there is a near perfect matching $F$ in $G$ such that $M \subseteq F$ and the $F$-unsaturated vertex $v$ is in $\Gamma_{G'}(x)$. So $F \cup \{xv\}$ is a perfect matching in $G'$ containing $M$.

Now we prove the sufficiency. Assume $G'$ is $n$-extendable. Select any matching $M''$ of size $n$ in $G$. Then there is a perfect matching $F'$ in $G'$ containing $M''$ and hence there is a vertex $u$ in $G$ such that $xu \in F'$. Clearly, $F' \backslash \{xu\}$ is a near perfect matching in $G$ containing $M''$. □

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