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A Note on γ-Graphs

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Abstract

As introduced in the paper by Fricke et. al., given a graph $G = (V, E)$, the γ-graph $G(\gamma) = (V(\gamma), E(\gamma))$ is the graph whose vertex set corresponds 1-to-1 with the γ-sets, or minimum-cardinality dominating sets, of G. Two γ-sets, say S_1 and S_2, are adjacent in $E(\gamma)$ if there exists a vertex $v \in S_1$ and a vertex $w \in S_2$ such that v is adjacent to w and $S_1 = S_2 - \{w\} \cup \{v\}$, or equivalently, $S_2 = S_1 - \{v\} \cup \{w\}$. In this paper we investigate two open questions regarding these γ-graphs. First, we consider whether every graph H is the γ-graph of some graph G, and we show that for every graph H, there exists a graph G such that $G(\gamma) \simeq H$. Second, we investigate when $G(\gamma)$ is disconnected. We prove that all graphs of order $n \leq 5$ have connected γ-graphs, and we determine all graphs G on six vertices for which $G(\gamma)$ is disconnected.

Keywords: dominating sets, γ-graphs.

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1. Introduction

Let $G = (V, E)$ be a graph with vertex set $V = \{v_1, v_2, \ldots, v_n\}$ and *order* $n = |V|$. We assume the reader is familiar with some basic terms in graph theory, specifically regarding domination. We briefly describe a few terms here, but refer the reader to [2, 3]. A set $S \subseteq V$ is a *dominating set* of G if every vertex in $V - S$ is adjacent to at least one vertex in S, or equivalently if the closed neighborhood $N[S]$ of S satisfies $N[S] = V$. The *domination number* $\gamma(G)$ of G equals the minimum cardinality of a dominating set S in G; we say that such a set S is a γ-*set*. We say that a vertex $v \in S$ has a *private neighbor* *with respect to* S if $N[v] - N[S - \{v\}] \neq \emptyset$, in which case every vertex in $N[v] - N[S - \{v\}]$ is called a *private neighbor* of v with respect to S. A vertex $w \in V - S$ is said to be an *external private neighbor*, or *epn*, of a vertex $v \in S$ if $N(w) \cap S = \{v\}$. If vertex $v \in S$ is not adjacent to any vertex in S it is called *its own private neighbor, or self-private neighbor, spn.*

As introduced in Fricke, et.al. [1], the γ*-graph* $G(\gamma) = (V(\gamma), E(\gamma))$ of a graph G is the graph whose vertices $V(\gamma)$ correspond 1-to-1 with the γ-sets of G, and two γ-sets, say S_1 and S_2, form an edge in $E(\gamma)$ if there exists a vertex $v \in S_1$ and a vertex $w \in S_2$ such that (i) v is adjacent to w and (ii) $S_1 = S_2 - \{w\} \cup \{v\}$ and $S_2 = S_1 - \{v\} \cup \{w\}$. We say that the γ-graph $G(\gamma)$ of graph G is *isomorphic* to a graph H, that is, $G(\gamma) \simeq H$ if and only if there exists a bijection $f : V(G(\gamma)) \to V(H)$ such that two vertices u, v are adjacent in $G(\gamma)$ if and only if $f(u)$ and $f(v)$ are adjacent in H.

We are interested in investigating the following open questions as described in [1].

1. Is it possible to construct a graph H that is not a γ-graph of any graph G?

2. Under what conditions is $G(\gamma)$ a disconnected graph?

Regarding the first question, we will show in Section 2 that for every graph H, there exists a graph G such that $G(\gamma) \simeq H$. Regarding the second question, we will show in Section 3 that the γ-graphs of all graphs of order $n \leq 5$ are connected. We further identify all graphs of order $n = 6$ having disconnected γ-graphs.

2. γ-Graph Existence

In the main result of this section, given any graph H, we show that there exists a graph G where $G(\gamma) \simeq H$. We begin with a couple of lemmas useful in proving this result.

Lemma 2.1. *Let $G = (V, E)$ be a graph and let $S \subset V$ be any γ-set of G. Let $v \in S$ be any vertex in S such that $N[S - \{v\}] = V - \{x\}$, that is $S - \{v\}$ dominates every vertex in V except one vertex x. Then the subgraph $G[N[x]]$ induced by the closed neighborhood $N[x]$ of x is a subgraph of $G(\gamma)$.*

Proof. If $S - \{v\}$ dominates every vertex except x, then $S - \{v\} \cup \{u\}$ is a γ-set for every vertex $u \in N[x]$. □

Corollary 2.2. *Let $G = (V, E)$ be a graph having a γ-set $S \subset V$ with the property that for every vertex $v \in S$, there does not exist an adjacent vertex $w \in V - S$ such that $S - \{v\} \cup \{w\}$ is a γ-set. Then S is an isolated vertex in $G(\gamma)$.*

An obvious generalization of Lemma 2.1 is the following.

Lemma 2.3. *Let $G = (V, E)$ be a graph and let $S \subset V$ be any γ-set of G. Let $v \in S$ be any vertex in S such that $N[S - \{v\}] = V - A$, that is $S - \{v\}$ dominates every vertex in V except vertices in A. Let $CN[A] = \cap_{u \in A} N[u]$ be the common neighborhood of the vertices in A. Then the induced subgraph $G[CN[A]]$ is a subgraph of $G(\gamma)$.*

Using the above results, we can prove the main theorem of this section.

Theorem 2.4. *For any graph H, there exists a graph G such that $G(\gamma) \simeq H$.*

Proof. Let $H = (V, E)$ be a graph where $V(H) = \{v_1, v_2, ..., v_n\}$. We show, by construction, that there exists graph G such that $G(\gamma) \simeq H$. We construct the graph G using the existing graph H. To form G, add a path P_3 of order 3 with vertices p_1, p_2, p_3 and add n edges of the form (v_i, p_2), for $1 \leq i \leq n$. Further, add two nonadjacent vertices x and y with edges (v_i, x) and (v_i, y), for $1 \leq i \leq n$. Note that since no vertex is adjacent to every other vertex, $\gamma(G) \geq 2$.

It is clear that $X_i = \{p_2, v_i\}$, $1 \leq i \leq n$, is a γ-set for G since p_2 dominates each vertex in the added P_3 and each vertex v_i, $1 \leq i \leq n$, in the original graph H. Further, each vertex v_i, for $1 \leq i \leq n$, dominates both x and y. Since no dominating set of G can contain less than two vertices, each X_i, $1 \leq i \leq n$, is a minimum-cardinality dominating set. To show there are no others, let S be a γ-set of G. Set S must contain p_2 otherwise S would have to contain both p_1, p_3 and some vertex $v_j \in V(H)$, thus making $|S| > 2$. Further, if $x \in S$ (or likewise $y \in S$), then since x and p_2 do not dominate y (or likewise x), there must exist another vertex in the dominating set, contradicting the minimality of S. This implies that $S = X_i$, for some $1 \leq i \leq n$. Hence, the sets X_i, $1 \leq i \leq n$, are the only minimum dominating sets for G.

Each γ-set differs by only one vertex as p_2 appears in every γ-set of G. Further, each vertex in $V(H)$ is adjacent to vertices x and y. Hence, the common neighborhood of x and y is $V(H)$. The result then follows by Lemma 2.3. □

Figure 1 shows the construction of the graph G from a graph H so that $G(\gamma) \simeq H$.

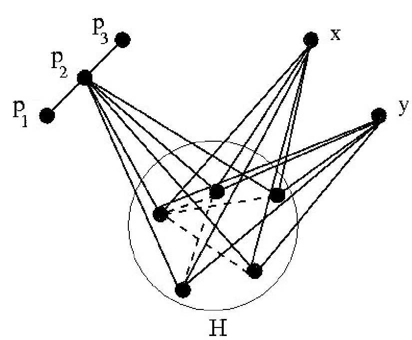

Figure 1: Construction of Graph G from Graph H

Note that as long as $\gamma(H) \geq 2$, we can reduce the size of the constructed graph, call it G', by adding a path P_2 of order 2 instead of order 3, where $V(P_2) = \{p_1, p_2\}$ and still add edges (v_i, p_2), for all $1 \leq i \leq n$. In this case we still add vertices x and y with corresponding edges (x, v_i) and (y, v_i), $1 \leq i \leq n$. This leads to the following proposition.

Proposition 2.5. *If $\gamma(H) \geq 2$, then the aforementioned constructed graph G' is such that $G'(\gamma) \simeq H$.*

Two immediate corollaries of Theorem 2.4 are as follows.

Corollary 2.6. *Every chordal graph is the γ-graph of a chordal graph.*

Proof. If a graph H is chordal, i.e. contains no chordless cycles, then so is the graph G constructed from H in Theorem 2.4. □

Corollary 2.7. *Every graph H of order n is the γ-graph of a graph G of order $n + 5$ (or $n + 4$ if $\gamma(H) \geq 2$) that contains H as a proper, induced subgraph.*

One interesting question that follows from Corollary 2.7 is: when is a graph isomorphic to its γ-graph? For complete graphs, we can state the following proposition.

Proposition 2.8. *Let $G = (V, E)$ be a connected graph of order n. Then $G(\gamma)$ is isomorphic to the complete graph K_r if and only if G has r vertices of degree $n - 1$.*

Thus, complete graphs are isomorphic to their γ-graphs. As noted in [1], C_{3k+2}, $k \geq 1$, the cycle on $3k+2$ vertices, is also isomorphic to $C_{3k+2}(\gamma)$. Further, we have found another class of graphs that are isomorphic to their γ-graphs. These graphs are combinations of cycles and complete graphs. Consider C_k, the cycle with k vertices $\{x_1, x_2, \ldots, x_k\}$. If k is odd, we replace each edge $(x_i, x_{(i+1) \bmod (k)}) \in E(C_k)$, $1 \leq i \leq k$, with a complete graph of size K_n. That is, we add vertices a_1, a_2,$\ldots$, a_{n-2} and all possible edges corresponding to these vertices and x_i and x_{i+1}. This is repeated for each of the original edges in C_k. If k is even, we replace one vertex x_1 with a complete graph K_n and add edges from x_k and x_2 to each vertex in the added K_n. Then, for each of the edges $(x_i, x_{i+1 \bmod (k)})$, $2 \leq i \leq k - 1$, we make the same replacement as we did when k was odd. We call the graph formed in this manner $K_n \cdot C_k$. Figure 2 shows $K_4 \cdot C_3$.

It is easy to show the following.

Proposition 2.9. $(K_n \cdot C_k)(\gamma) \simeq K_n \cdot C_k$.

Proof. We only present the case where k is odd. Since the graph $(K_n \cdot C_k)$ consists of k K_n-subgraphs arranged along an odd cycle C_k, we have to choose vertices that will dominate the vertices in each K_n subgraph. This is minimally accomplished by choosing vertices that are on the "inner" cycle C_k. Each of these vertices dominate two adjacent K_n-subgraphs. So choosing every other of these vertices dominates all but one of the K_n-subgraphs. Since k is odd, this leaves one K_n-subgraph undominated. Any of the vertices

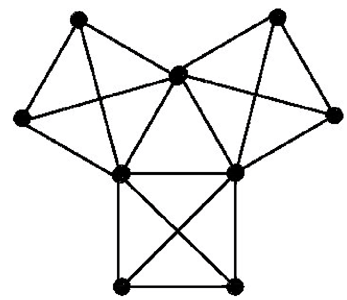

Figure 2: The Graph of $K_4 \cdot C_3$

of the undominated K_n can be used to complete the dominating set and thus create a K_n-subgraph in $(K_n \cdot C_k)(\gamma)$. Since a dominating set can be built like this starting from any vertex on the cycle C_k, it is clear that $(K_n \cdot C_k)(\gamma) \simeq (K_n \cdot C_k)$. □

The question of finding graphs that are isomorphic to their γ-graphs seems to be difficult to answer. We leave this as an open problem.

3. γ-Graph Connectivity

A second question from [1] was that of finding conditions under which the γ-graph of a graph is disconnected. The following proposition shows that if G is disconnected, then its γ-graph $G(\gamma)$ is connected.

Proposition 3.1. *Let G be a graph consisting of two connected components G_1 and G_2. Then $G(\gamma) \simeq (G_1(\gamma) \Box G_2(\gamma))$ is connected, where $\Box$ denotes the Cartesian product of two graphs.*

Corollary 3.2. *If a graph G is disconnected, then $G(\gamma)$ is connected.*

Corollary 3.3. *Let $G = (V, E)$ be a graph having an isolated vertex x. Then $G(\gamma) \simeq (G - \{x\})(\gamma)$.*

Next, we show that all graphs of order $n \leq 5$ have connected γ-graphs, and we exhibit all graphs of order $n = 6$ which have disconnected γ-graphs. First we need the following lemma.

Lemma 3.4. *Let $G = (V, E)$ be a graph. If γ_1 and γ_2 are γ-sets of G such that $\gamma_1 \cap \gamma_2 = \emptyset$, then the induced subgraph $G[\gamma_1 \cup \gamma_2]$ contains a matching between the vertices in γ_1 and the vertices in γ_2 in each of the following cases.*

1. *$\gamma(G) = 2$, or*
2. *$\gamma_1 \cup \gamma_2 = V(G)$.*

Proof. In the first case where $\gamma(G) = 2$, since γ_1 is a dominating set, each vertex in γ_2 is adjacent to some vertex in γ_1. Since γ_2 is a dominating set, each vertex in γ_1 is adjacent to some vertex in γ_2. Since $|\gamma_1| = |\gamma_2| = 2$ and $\gamma_1 \cap \gamma_2 = \emptyset$, the result follows.

In the second case where $\gamma_1 \cup \gamma_2 = V(G)$, suppose there is a set $A \subseteq \gamma_1$ such that $|N(A) \cap \gamma_2| < |A|$, where $N(A)$ refers to the open neighborhood of A. Now consider the set $B = (N(A) \cap \gamma_2) \cup (\gamma_1 - A)$. Since γ_1 is a dominating set, $\gamma_1 - A$ dominates $\gamma_2 - (N(A) \cap \gamma_2)$ and $(N(A) \cap \gamma_2)$ dominates A. Hence B is a dominating set of smaller cardinality than γ_1, a contradiction. Thus each subset $A \subseteq \gamma_1$ has $|N(A) \cap \gamma_2| \geq |A|$, and the result follows from Hall's Marriage Theorem [4]. □

Theorem 3.5. *If $G = (V, E)$ is a graph of order $n \leq 5$, then $G(\gamma)$ is connected.*

Proof. Let G be a graph of order $n \leq 5$. We must show that $G(\gamma)$ is connected. From Corollary 3.2 we know that if G is not connected then $G(\gamma)$ is connected. Let us therefore assume that G is connected. If $\gamma(G) = 1$, then $G(\gamma)$ is a complete graph, and therefore is connected.

So, we can assume that $\gamma(G) \geq 2$. But since we can assume that G is connected, we know that $\gamma(G) \leq \lfloor n/2 \rfloor$. Therefore, we can assume that $\gamma(G) = 2$. We can also assume that $4 \leq n \leq 5$, since there are only four graphs of order $n = 3$ and one can easily see that each of these four graphs has a γ-graph that is a complete graph, and is therefore connected.

Suppose next that there exist distinct γ-sets $\gamma_1 = \{x_1, y_1\}$ and $\gamma_2 = \{x_2, y_2\}$ such that $\gamma_1 \neq \gamma_2$ and there is no path in $G(\gamma)$ between the vertices corresponding to γ_1 and γ_2, say v_1 and v_2, respectively. Since γ_1 and γ_2 are distinct, without loss of generality suppose $x_1 \neq x_2$, $x_1 \neq y_2$, and $x_2 \neq y_1$. Thus, either $y_1 = y_2$ or $y_1 \neq y_2$.

If $y_1 = y_2 = y$, then $(x_1, x_2) \notin E(G)$ for otherwise there would be a direct v_1-to-v_2-path in $G(\gamma)$. This implies that (y, x_1) and (y, x_2) must exist in $E(G)$. Hence, there exists a vertex, call it $z \in V(G) - \gamma_1 - \gamma_2$ such that $(y, z) \notin E(G)$ but $(x_1, z), (x_2, z) \in E(G)$ since $x_1 \in \gamma_1$ and $x_2 \in \gamma_2$ and x_1 and x_2 are not their own private neighbors relative to γ_1 and γ_2. If $n = 4$, then $\gamma = \{y, z\}$ is a γ-set, and thus $\{v_1, v, v_2\}$, corresponding to γ-sets $\gamma_1, \gamma, \gamma_2$ respectively, is a v_1-to-v_2-path in $G(\gamma)$. Hence $n = 5$. If $n = 5$, though, then there exists a vertex $w \neq z \in V - \gamma_1 - \gamma_2$. However, if $(y, w) \in E(G)$, then $\gamma = \{y, z\}$ is a γ-set of G and as before supplies a v_1-to-v_2-path in $G(\gamma)$. Otherwise, $(x_1, w), (x_2, w) \in E(G)$. However, then $\{x_1, x_2\}$ is a γ-set of G and supplies a v_1-to-v_2-path in $G(\gamma)$. As this exhausts every option, if $y_1 = y_2$, then $G(\gamma)$ is connected.

Now consider the case where $y_1 \neq y_2$. This implies that $\gamma_1 \cap \gamma_2 = \emptyset$. By Lemma 3.4, there exists a matching between the vertices of γ_1 and γ_2. Without loss of generality, let us assume $(x_1, x_2), (y_1, y_2) \in E(G)$. If $n = 4$, then since $\gamma = \{x_1, y_2\}$ is a γ-set, there exists a v_1-to-v_2-path in $G(\gamma)$. If $n = 5$, there exists a vertex $z \in V(G) - \gamma_1 - \gamma_2$ which is adjacent to at least one vertex in each of γ_1 and γ_2. Without loss of generality, suppose $(x_1, z) \in E(G)$. Then, there is a v_1-to-v_2-path in $G(\gamma)$ corresponding to the γ-sets γ_1, $\gamma = \{x_1, y_2\}$, and γ_2. Thus, if $y_1 \neq y_2$, we have found that $G(\gamma)$ is connected. □

For the case of a graph with six vertices we can characterize the graphs whose γ-graphs are disconnected. To this end, suppose $G = (V, E)$ is a graph with $|V| = 6$ such that $G(\gamma)$ is disconnected. From Corollary 3.2, we can assume that G is connected. Since $G(\gamma)$ is disconnected there exists vertices $v_1, v_2 \in V(G(\gamma))$ such that there does not exist a v_1-to-v_2 path in $G(\gamma)$. Suppose that γ_1 and γ_2 are the γ-sets of G corresponding to vertices v_1 and v_2. As in Theorem 3.5, note that if $\gamma(G) = 1$, there cannot be two γ-sets that are disjoint since each vertex in a γ-set must be adjacent to all other vertices. Hence $\gamma(G) \geq 2$. If $\gamma(G) = 3 = \lfloor \frac{n}{2} \rfloor$, Lemma 3.4 guarantees that there is a matching of the elements of γ_1 and γ_2 thus supplying a v_1-to-v_2-path in $G(\gamma)$. Hence $\gamma(G) = 2$. We now break our discussion into two cases depending on whether (A) $\gamma_1 \cap \gamma_2 = \emptyset$ or (B) $\gamma_1 \cap \gamma_2 \neq \emptyset$.

Let us first consider Case (A). Let $\gamma_1 = \{x_1, y_1\}$ and $\gamma_2 = \{x_2, y_2\}$. Since $\gamma_1 \cap \gamma_2 = \emptyset$, each of x_1, x_2, y_1, y_2 is distinct. Further, by Lemma 3.4, there exists a matching between the vertices in γ_1 and γ_2, say this matching is (x_1, x_2) and (y_1, y_2). Suppose there exists a vertex of γ_1 (or γ_2), say x_1, that is adjacent to each vertex of $V - \gamma_1 - \gamma_2 = \{z_1, z_2\}$. Then there is a v_1-to-v_2-path in $G(\gamma)$, namely $\{v_1, v, v_2\}$ corresponding to γ-sets γ_1, $\gamma = \{x_1, y_2\}$, and γ_2 respectively. So no vertex in γ_1 or γ_2 can be adjacent to each vertex of $V - \gamma_1 - \gamma_2$. This implies that x_1 and y_1 must be adjacent to exactly one vertex in $V - \gamma_1 - \gamma_2$, say $(x_1, z_1), (y_1, z_2) \in E(G)$. Note that since $(x_1, x_2), (y_1, y_2) \in E(G)$, if (x_2, z_1) and (y_2, z_2) are edges in $E(G)$, there exists a v_1-to-v_2 path in $G(\gamma)$ since both $\{x_2, y_1\}$ and $\{x_1, y_2\}$ are γ-sets. Hence $(x_2, z_1), (y_2, z_2) \notin E(G)$, but rather $(x_2, z_2), (y_2, z_1) \in E(G)$ since γ_2 is a dominating set and no vertex in γ_2 can be adjacent to both z_1 and z_2. Also, note that the edges (x_1, y_2) and (y_1, x_2) cannot be present if the γ-graph is to be disconnected. To see this, suppose G contains the edge (x_1, y_2). Then there is a v_1-to-v_2-path in $G(\gamma)$ through the vertices corresponding to the sequence of γ-sets $\{x_1, y_1\}$, $\{x_1, z_2\}$, $\{z_1, z_2\}$, $\{z_2, y_2\}$, $\{x_2, y_2\}$. The case where $(y_1, x_2) \in E(G)$ is the same by symmetry. This leaves the only possible graph satisfying Case (A) to be C_6, the cycle of order 6, as shown in Case (A) of Figure 3. The edges (x_1, y_1), (x_2, y_2), and (z_1, z_2) may or may not be present in a Case (A) graph without adding a v_1-to-v_2-path in $G(\gamma)$.

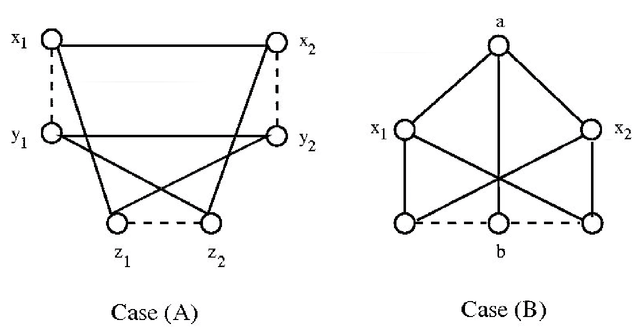

Figure 3: Graphs of Order 6 with Disconnected γ-Graphs

Now, let us consider Case (B). Since $\gamma(G) = 2$, $|\gamma_1 \cap \gamma_2| = 1$, and call this common vertex a. There exists two nonadjacent vertices x_1 and x_2 so that $\{a, x_1\} = \gamma_1$ and $\{a, x_2\} = \gamma_2$. Note that $(x_1, x_2) \notin E(G)$ for otherwise there would be a path between v_1 and v_2 corresponding to γ_1 and γ_2 in $G(\gamma)$. This implies that $(a, x_1), (a, x_2) \in E(G)$. Vertex a must have a private neighbor in both γ_1 and γ_2. Thus it must be adjacent to at least one vertex, call it b, in $V - \gamma_1 - \gamma_2$, otherwise $\{x_1, x_2\}$ is a γ-set and thus creates a path in $G(\gamma)$ between v_1 and v_2. However a cannot be adjacent to more than one vertex in $V - \gamma_1 - \gamma_2$. To see this, if a is adjacent to all three vertices in $V - \gamma_1 - \gamma_2$ then $\{a\}$ is a dominating set, contradicting $\gamma(G) = 2$. Further, if a is adjacent to two of the three vertices in $V - \gamma_1 - \gamma_2$ and not adjacent to, say, vertex $z \in V - \gamma_1 - \gamma_2$, then $(x_1, z), (x_2, z) \in E(G)$. This implies that $\{a, z\}$ is a γ-set and thus supplies a v_1-to-v_2-path in $G(\gamma)$. Now, since γ_1 and γ_2 are dominating sets, we must, then, have edges connecting x_1 and x_2 to the two remaining vertices in $V - \gamma_1 - \gamma_2$. Hence, the created graph in Case B (with possible added edges from b to each of the other vertices in $V - \gamma_1 - \gamma_2$, as shown in Case (B) of Figure 3) is the only graph on six vertices with two γ sets, γ_1 and γ_2 with $\gamma_1 \cap \gamma_2 \neq \emptyset$ with a disconnected γ-graph. We have thus shown Theorem 3.6.

Theorem 3.6. *The only graphs of order $n = 6$ with disconnected γ-graphs are the graphs in Case (A) and the graphs in Case (B).*

Note that the construction proofs for Theorem 2.4 and Proposition 2.5 can be used to construct disconnected γ-graphs. In fact, the Case (B) graph is a construction of the form indicated in Proposition 2.5 where we have taken two nonadjacent vertices x_1 and x_2 and added a path of length 2 with vertices a and b in which vertex a is adjacent to x_1 and x_2 as well as two nonadjacent vertices y_1 and y_2 which are adjacent to each of x_1 and x_2. Figure 4 represents the Case (B) graph in a way to see this construction more clearly.

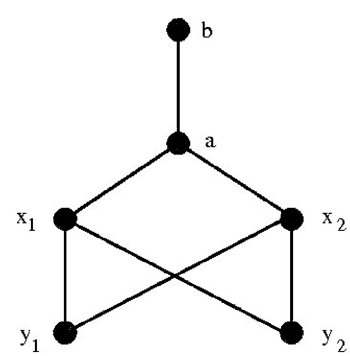

Figure 4: Case (B)

4. Open Questions

Both in the proof here that every graph is the γ-graph of some graph and in the proof in [1] that every tree is the γ-graph of some graph, there are constructions of graphs where there are graphs H whose γ-graphs are isomorphic to an induced subgraph of H. We might ask for which graphs H are the γ -graphs of a graph G that is smaller than H? We have several examples of this. For example, the γ-graph of the four-cycle C_4 is of order six. Further all n-cubes, having order 2^n are the γ-graphs of coronas of order $2n$. Also, the question of which graphs are isomorphic to their γ-graphs seems particularily intriguing.

Added In Proof. The authors have recently become aware of the existence of the 2008 paper "γ-graph of a graph" by K. Subramanian and N. Sridharan appearing in *Bull. Kerala Math. Assoc.* **5**(1), pp. 17-34. Two other papers on this topic also exist: N. Sridharan and K. Subramanian, Trees and unicyclic graphs are γ-graphs, *J. Combin. Math. Combin. Comput.*, **69** (2009), 231-236, and S. A. Lakshmanan and A. Vijayakumar, The Gamma Graph of a Graph, *AKCE Int. J. Graphs Comb.*, **7**(1), 2010, pp. 53-59. It is important to note that in these papers the definition of γ-graphs is different from ours, and thus these are two different classes of graphs.

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