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On the Regularity of Some Signed Graph Structures *

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Abstract

A *signed graph* (or *sigraph* in short) is an ordered pair $S = (S^u, \sigma)$, where S^u is a graph $G = (V, E)$ and $\sigma : E \to \{+, -\}$ is a function from the edge set E of S^u into the set $\{+, -\}$. A sigraph S is called *signed-regular* if the number of positive edges, $d^+(v)$ incident at a vertex v in S, is independent of the choice of v in S and the number of negative edges, $d^-(v)$ incident at a vertex v in S is also independent of the choice of v in S. In this paper, we characterize the signed-regularity of line sigraph, $\times$-line sigraph and total sigraph.

Keywords: Sigraph, line sigraph, $\times$-line sigraph, total sigraph.

2010 Mathematics Subject Classification: *Primary:* 05C 22; *Secondary:* 05C 75.

1. Introduction

For standard terminology and notation in graph theory we refer Harary [5], West [9] and for sigraphs we refer Zaslavsky [10], [11]. A *signed graph* (or *sigraph* in short) is an ordered pair $S = (S^u, \sigma)$, where S^u is a graph $G = (V, E)$, called the *underlying graph* of S and $\sigma : E \to \{+, -\}$ is a function from the edge set E of S^u into the set $\{+, -\}$, called the *signature* of S. Let $E^+(S) = \{e \in E(G) : \sigma(e) = +\}$ and $E^-(S) = \{e \in E(G) : \sigma(e) = -\}$. The elements of $E^+(S)$ and $E^-(S)$ are called *positive* and *negative* edges of S, respectively. A sigraph is said to be *homogeneous* if all its edges have the same sign and *heterogeneous* otherwise.

Two edges of a graph are said to be *adjacent* if they are incident with a common vertex. A set of edges in a graph is *independent* if no two of them are adjacent. A graph is called *r-regular* if all its vertices are of degree r. A bipartite graph is called (r_1, r_2)-*semiregular* if its vertex bipartition $V = V_1 \cup V_2$ has all vertices in V_i of degree r_i for $i = 1, 2$. A sigraph S is called *signed-regular* if the number of positive edges, $d^+(v)$ incident at a vertex v in

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S, is independent of the choice of v in S and the number of negative edges, $d^-(v)$ incident at a vertex v in S is also independent of the choice of v in S, i.e. S is (i,j)-signed-regular, where $i = d^+(v)$ is the positive degree of v in S and $j = d^-(v)$ is the negative degree of v in S. The *negation* $\eta(S)$ of a sigraph S is a sigraph obtained from S by negating the sign of every edge of S, that means to find $\eta(S)$ we change the sign of every edge to its opposite in S.

A *u-v path* in a sigraph S is an alternating sequence of vertices and edges $u, e_1, v_1, e_2, ..., e_n, v$, beginning and ending with vertices, in which each edge is incident with the two vertices immediately preceding and following it and no vertex is repeated. A path is called *homogeneous* if all its edges have the same sign and *heterogeneous* otherwise. The *adjacency matrix* $A(S)$ of a sigraph S of order n is an $n \times n$ matrix in which $a_{ij} = \sigma(v_i v_j)$ if v_i and v_j are adjacent, and 0 if they are not. The eigenvalues of the adjacency matrix of S are called the *eigenvalues* of S. The *energy* $E'(S)$ of a sigraph S is the sum of absolute values of the eigenvalues of S.

For a sigraph S, Behzad and Chartrand [4] defined its *line sigraph* $L(S)$ as the sigraph in which the edges of S are represented as vertices, two of these vertices are defined adjacent whenever the corresponding edges in S have a vertex in common, any such edge ef is defined to be negative whenever both e and f are negative edges in S. The second iterated line sigraph of S is then defined recursively as $L^2(S) = L(L(S))$.

In [6], the author introduced a variation of the above standard notion of $L(S)$ as follows: it is a sigraph denoted by $L_\times(S)$ and defined on the line graph $L(S^u)$ of the graph S^u by assigning to each edge ef of $L(S^u)$, the product of signs of the adjacent edges e and f in S. $L_\times(S)$ is called the $\times$*-line sigraph* of S. In [1], the author found many properties of $\times$-line sigraph.

A *marked signed graph* is an ordered pair $S_\mu = (S, \mu)$ where $S = (S^u, \sigma)$ is a sigraph and $\mu : V(S^u) \to \{+, -\}$ is a function from the vertex set $V(S^u)$ of S^u into the set $\{+, -\}$, called a *marking* of S. The marking μ_σ defined by

$$\mu_\sigma(v) = \prod_{e_j \in E_v} \sigma(e_j), v \in V(S),$$

is called the *canonical marking* of S, where E_v is set of edges e_j incident at v in S.

The *total graph* [2] $T(G)$ of a graph G is that graph whose vertex set is $V(G) \cup E(G)$ where $V(G)$ and $E(G)$ are vertex set and edge set of G, respectively and in $T(G)$ two vertices are adjacent if and only if they are adjacent or incident in G. If all the vertices of $T(G)$ have equal degree, then it is said to be a *regular total graph*.

Let $S = (V, E, \sigma)$ be any sigraph. Its *total sigraph* $T(S)$ [Figure 1] has $T(S^u)$ as its underlying graph and for any edge uv of $T(S^u)$

$$\sigma_T(uv) = \begin{cases} \sigma(uv) & \text{if } u, v \in V \\ \sigma(u)\sigma(v) & \text{if } u, v \in E \\ \sigma(u) \prod_{e_j \in E_v} \sigma(e_j) & \text{if } u \in E \text{ and } v \in V. \end{cases}$$

We observe that a sigraph S and its $\times$-line sigraph $L_\times(S)$ are induced subsigraphs of the total sigraph $T(S)$ of S.

Throughout the text, without loss of generality we consider connected sigraphs, since disconnected components can be dealt separately in the same manner.

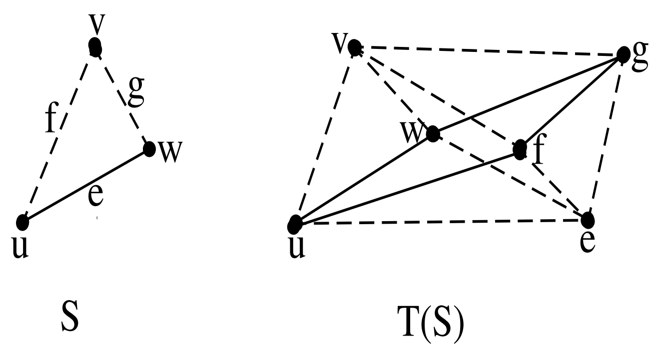

Figure 1: Total sigraph of a sigraph.

2. Signed-regularity of line sigraphs

It is given in [8] that the line graph $L(G)$ of a connected graph G is regular if and only if G is either regular or (r_1, r_2)-semiregular. It can be easily verified that this result works for disconnected graph G, also. If G is a graph of order n and regular of degree r, then its line graph $L(G)$ is a regular graph of order $n_L = \frac{nr}{2}$ and of degree $r_L = 2(r-1)$. If G is (r_1, r_2)-semiregular, then its line graph $L(G)$ is a regular graph of degree $r_L = r_1 + r_2 - 2$. In this section, we obtain a characterization of signed-regularity of line sigraph of a sigraph.

Theorem 1. *The line sigraph $L(S)$ of a sigraph $S = (S^u, \sigma)$, where S^u is either regular or semiregular, is signed-regular if and only if S is all-negative or $E^-(S)$ is an independent set of edges of S.*

Proof. **Necessity**: Suppose $L(S)$ is the signed-regular sigraph of a sigraph S. Suppose $L(S)$ is all-negative, then S is all-negative. So, let $L(S)$ is either all-positive or heterogeneous sigraph. Let if possible, $E^-(S)$ is not an independent set of edges of S, then $E^-(S)$ contains at least two adjacent edges e and f. Now, by the definition of $L(S)$, the negative degree of every vertex in $L(S)$ which corresponds to the positive edge in S, is zero. But the negative degree of the vertex e in $L(S)$ is at least one. That means, $L(S)$ is not signed-regular, a contradiction to the assumption. Thus, by the contraposition, result follows.

Sufficiency: Suppose $S = (S^u, \sigma)$ is a sigraph, where S^u is either regular or semiregular, that means $L(S^u)$ is regular. If S is all-negative, then $L(S)$ is an all-negative regular sigraph. Now, suppose $E^{-}(S)$ is an independent set of edges of S, then $L(S)$ is an all-positive signed-regular sigraph. Hence the theorem. $\square$

Example 2. Two sigraphs whose line sigraph is signed-regular, are shown in Figure 2.

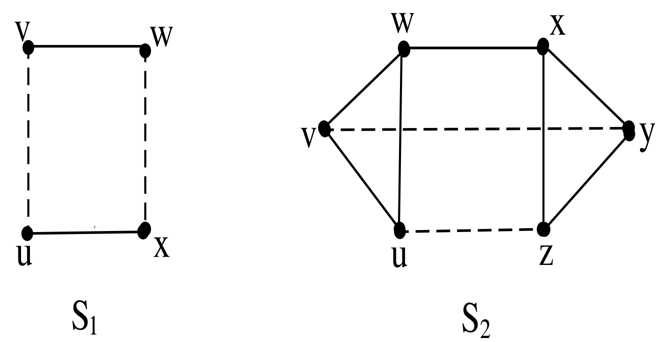

Figure 2: Sigraphs whose line sigraph is signed-regular.

Corollary 3. *The line sigraph $L(S)$ of a heterogeneous (i, j)-signed-regular sigraph S is signed-regular if and only if $j = 1$.*

Remark 4. Suppose $S = (S^u, \sigma)$, where S^u is regular or semiregular, is a sigraph. If $E^{-}(S)$ is an independent set of edges of S, then $L(S)$ is $L(S^u)$ with all-positive signs.

Theorem 5. [7] *If G is regular graph of order n and of degree $r \geq 3$, then $L^2(G)$ has exactly $nr(r-2)/2$ negative eigenvalues, all being equal to -2 and energy $E'(L^2(G)) = 2nr(r-2)$.*

Remark 6. Suppose $S = (S^u, \sigma)$, where S^u is a regular graph of order n and of degree $r \geq 3$, is a sigraph. If $E^{-}(S)$ is an independent set of edges of S, then $L^2(S)$ has exactly $nr(r-2)/2$ negative eigenvalues, all being equal to -2 and energy $E'(L^2(S)) = 2nr(r-2)$.

3. Signed-regularity of $\times$-line sigraphs

In this section, we find a characterization of signed-regularity of $\times$-line sigraph of a sigraph.

Theorem 7. *If $S = (S^u, \sigma)$ is a homogeneous sigraph, then its $\times$-line sigraph $L_\times(S)$ is signed-regular if and only if S^u is regular or semiregular.*

Proof. Suppose $L_\times(S)$ is signed-regular, then $(L(S))^u$ is regular. Which implies that S^u is regular or semiregular. Next, suppose S^u is regular or semiregular, then $(L(S))^u$ is regular. Since S is homogeneous, therefore $L_\times(S)$ is all-positive sigraph. Thus, $L_\times(S)$ is an all-positive signed-regular sigraph. $\square$

Theorem 8. *The $\times$-line sigraph $L_\times(S)$ of a heterogeneous sigraph $S = (S^u, \sigma)$, where S^u is regular or semiregular, is signed-regular if and only if the following conditions hold in S:*

(i) *if u and w are joined by a homogeneous path of length two, then $d^+(u) = d^+(w)$ and $d^-(u) = d^-(w)$ and*

(ii) *if two vertices u and w in S are joined by a heterogeneous path of length two through v, then $d^+(v) - d^-(v) = d^-(w) - d^+(u)$.*

Proof. **Necessity**: Suppose $L_\times(S)$ is a signed-regular sigraph of a heterogeneous sigraph S. Let $e = uv$ and $f = vw$ are two positive edges of S. Since e and f are the vertices in $L_\times(S)$, therefore,

$$d^+_\times(e) = d^+(u) + d^+(v) - 2, \tag{1}$$

$$d^-_\times(e) = d^-(u) + d^-(v), \tag{2}$$

$$d^+_\times(f) = d^+(v) + d^+(w) - 2 \tag{3}$$

and

$$d^-_\times(f) = d^-(v) + d^-(w). \tag{4}$$

Since $L_\times(S)$ is signed-regular, therefore,

$$d^+_\times(e) = d^+_\times(f) \tag{5}$$

and

$$d^-_\times(e) = d^-_\times(f). \tag{6}$$

Now, using Eqs. (1), (3) and (5), $d^+(u) + d^+(v) - 2 = d^+(v) + d^+(w) - 2$. That implies, $d^+(u) = d^+(w)$. And using Eqs. (2), (4) and (6), $d^-(u) + d^-(v) = d^-(v) + d^-(w)$. That implies, $d^-(u) = d^-(w)$. Now, let $e = uv$ and $f = vw$ are two negative edges of S. Then,

$$d^+_\times(e) = d^-(u) + d^-(v) - 2, \tag{7}$$

$$d^-_\times(e) = d^+(u) + d^+(v), \tag{8}$$

$$d^+_\times(f) = d^-(v) + d^-(w) - 2 \tag{9}$$

and

$$d^-_\times(f) = d^+(v) + d^+(w). \tag{10}$$

Since $L_\times(S)$ is signed-regular, then using Eqs. (7) and (9), $d^-(u) = d^-(w)$. And using Eqs. (8) and (10), $d^+(u) = d^+(w)$. Thus, (i) follows.

Next, let $e = uv$ and $f = vw$ are positive and negative edges of S, respectively. Then,

$$d^+_\times(e) = d^+(u) + d^+(v) - 2, \tag{11}$$

$$d^-_\times(e) = d^-(u) + d^-(v), \tag{12}$$

$$d^+_\times(f) = d^-(v) + d^-(w) - 2 \tag{13}$$

and

$$d^-_\times(f) = d^+(v) + d^+(w). \tag{14}$$

Since $L_\times(S)$ is signed-regular, then using Eqs. (11) and (13), $d^+(v) - d^-(v) = d^-(w) - d^+(u)$. And using Eqs. (12) and (14), $d^+(v) - d^-(v) = d^-(u) - d^+(w) = d^-(w) - d^+(u)$. Thus, (ii) follows.

Sufficiency: Suppose conditions (i) and (ii) hold for a given sigraph $S = (S^u, \sigma)$, where S^u is regular or semiregular. Clearly, $L(S^u)$ is regular. Then, we shall show that $L_\times(S)$ is signed-regular. Let $e = uv$ and $f = vw$ are two positive edges of S. Now, due to condition (i), Eqs. (1) and (3), $d^+_\times(e) = d^+_\times(f)$. And due to condition (i), Eqs. (2) and (4), $d^-_\times(e) = d^-_\times(f)$. Now, let $e = uv$ and $f = vw$ are two negative edges of S. Then, due to condition (i), Eqs. (7) and (9), $d^+_\times(e) = d^+_\times(f)$. And due to condition (i), Eqs. (8) and (10), $d^-_\times(e) = d^-_\times(f)$. Next, let $e = uv$ and $f = vw$ are positive and negative edges of S, respectively. Then, due to condition (ii), Eqs. (11) and (13), $d^+_\times(e) = d^+_\times(f)$. And due to condition (ii), Eqs. (12) and (14), $d^-_\times(e) = d^-_\times(f)$. Since $e = uv$ and $f = vw$ are arbitrary adjacent edges of S, therefore $L_\times(S)$ is signed-regular. □

Example 9. Two sigraphs whose $\times$-line sigraph is signed-regular, are shown in Figure 3.

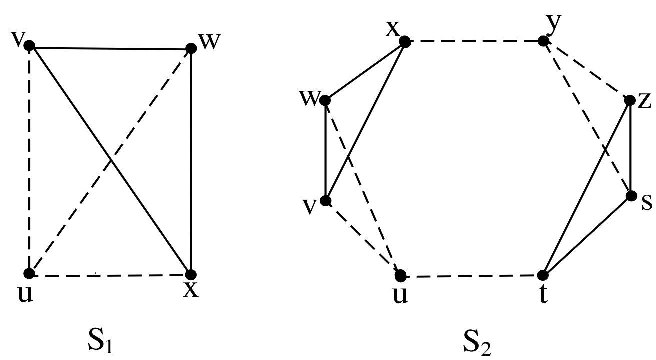

Figure 3: Sigraphs whose $\times$-line sigraph is signed-regular.

Theorem 10. *The $\times$-line sigraph $L_\times(S)$ of a heterogeneous (i, j)-signed-regular sigraph S is signed-regular if and only if $i = j$.*

Proof. Result follows from Theorem 8. □

Corollary 11. *If the $\times$-line sigraph $L_\times(S)$ of a sigraph $S = (S^u, \sigma)$, where S^u is regular or semiregular, is signed-regular, then $L_\times(\eta(S))$ is also signed-regular.*

Proof. For any sigraph $S, L_\times(S) = L_\times(\eta(S))$. Thus, if $L_\times(S)$ is (i, j)-signed-regular, then $L_\times(\eta(S))$ will also be (i, j)-signed-regular. □

4. Signed-regularity of total sigraphs

In the literature [3], it is available that for any graph G, its total graph $T(G)$ is regular if and only if G is regular. In this section, we give a characterization of signed-regularity of total sigraph of a sigraph.

Theorem 12. *If a heterogeneous sigraph $S = (S^u, \sigma)$, where S^u is regular, is not signed-regular, then its total sigraph $T(S)$ is not signed-regular.*

Proof. Suppose a heterogeneous sigraph S is not signed-regular. Let u and v are two vertices in S with $d^+(u) = i$, $d^-(u) = j$, $d^+(v) = k$ and $d^-(v) = l$. Now, we consider four cases:

Case(i): Suppose j and l both are even. Then,

$$d_T^+(u) = 2i, \tag{15}$$

$$d_T^-(u) = 2j, \tag{16}$$

$$d_T^+(v) = 2k \tag{17}$$

and

$$d_T^-(v) = 2l. \tag{18}$$

Suppose $T(S)$ is signed-regular, then from Eqs. (15) and (17), $i = k$ and from Eqs. (16) and (18), $j = l$. Thus S is signed-regular, a contradiction to the assumption. So this case is not possible.

Case(ii): Suppose j and l both are odd and $e = uv$ be a positive edge in S. Then,

$$d_T^+(u) = i + j, \tag{19}$$

$$d_T^-(u) = i + j, \tag{20}$$

$$d_T^+(v) = k + l, \tag{21}$$

$$d_T^-(v) = k + l, \tag{22}$$

$$d_T^+(e) = i + k - 2 \tag{23}$$

and

$$d_T^-(e) = j + l - 2. \tag{24}$$

Suppose $T(S)$ is signed-regular, then from Eqs. (19) and (23), $i + j = i + k - 2$ and from Eqs. (20) and (24), $i + j = j + l - 2$. Now adding these two, $i + j = k + l - 4$. But from Eqs. (19) and (21), $i + j = k + l$. So this case is not possible.

Case(iii): Suppose j is even and l is odd and $e = uv$ be a positive edge in S. Since S is heterogeneous sigraph, therefore without loss of generality, suppose $f = vw$ is a negative edge in S with $d^+(w) = m$, $d^-(w) = n$.

Subcase(a): When n is odd. Then,

$$d_T^+(u) = 2i, \tag{25}$$

$$d_T^-(u) = 2j, \tag{26}$$

$$d_T^+(v) = k + l, \tag{27}$$

$$d_T^-(v) = k + l, \tag{28}$$

$$d_T^+(e) = i + k - 1, \tag{29}$$

$$d_T^-(e) = j + l + 1, \tag{30}$$

$$d_T^+(f) = l + n \tag{31}$$

and

$$d_T^-(f) = k + m. \tag{32}$$

Suppose $T(S)$ is signed-regular, then from Eqs. (25), (26), (27) and (28), $i = j$. From Eqs. (25) and (29), $i = k - 1$ and from Eqs. (26) and (30), $j = l + 1$. From Eqs. (27) and (31), $k = n$ and from Eqs. (28) and (32), $l = m$. Thus, for a positive edge, the positive degree and the negative degree of its one end vertex are equal and the positive degree of the other end vertex is two more to the negative degree of this vertex. But there is at least a positive edge incident at w and the positive degree of w is two less to the negative degree of w. So this case is not possible.

Subcase(b): When n is even. Then,

$$d_T^+(u) = 2i, \tag{33}$$

$$d_T^-(u) = 2j, \tag{34}$$

$$d_T^+(v) = k + l, \tag{35}$$

$$d_T^-(v) = k + l, \tag{36}$$

$$d_T^+(w) = 2m, \tag{37}$$

$$d_T^-(w) = 2n, \tag{38}$$

$$d_T^+(e) = i + k - 1, \tag{39}$$

$$d_T^-(e) = j + l + 1, \tag{40}$$

$$d_T^+(f) = l + n - 1 \tag{41}$$

and

$$d_T^-(f) = k + m + 1. \tag{42}$$

Suppose $T(S)$ is signed-regular, then from Eqs. (33), (34), (35), (36), (37) and (38), $i = j = m = n$. From Eqs. (33) and (39), $i = k - 1$ and from Eqs. (38) and (42), $2n - m = k + 1$. Clearly, these two are absurd conditions being $n = m = i$. So this case is not possible.

Case(iv): Suppose j is even and l is odd, then due to arguments discussed in case(iii), we have this case is also not possible. From all the above cases theorem follows. □

Theorem 13. *The total sigraph $T(S)$ of an all-positive signed-regular sigraph $S = (S^u, \sigma)$ is signed-regular.*

Proof. Since S^u is regular, then $T(S^u)$ is regular. Now, if S is all-positive, then its $T(S)$ is all-positive. Thus, $T(S)$ is signed-regular. □

Theorem 14. *The total sigraph $T(S)$ of an all-negative $(0, j)$-signed-regular sigraph $S = (S^u, \sigma)$, where $j > 0$, is not signed-regular.*

Proof. If j is even, then the positive degree of the vertices of $T(S)$ which are corresponding to the vertices of S, is zero but the positive degree of the vertices of $T(S)$ which are corresponding to the edges of S, is at least two. Thus, $T(S)$ is not signed-regular. Next, if j is odd, then the negative degree of the vertices of $T(S)$ which are corresponding to the edges of S, is zero but the negative degree of the vertices of $T(S)$ which are corresponding to the vertices of S, is at least two. Thus, $T(S)$ is not signed-regular. □

Theorem 15. *The total sigraph $T(S)$ of a heterogeneous (i, j)-signed-regular sigraph S is signed-regular if and only if j is even and $i = j - 1$, where $i = d^+(v)$ and $j = d^-(v)$ for $v \in V(S)$.*

Proof. **Necessity**: Suppose $T(S)$ is a signed-regular sigraph for a heterogeneous (i, j)-signed-regular sigraph S with $i = d^+(v)$ and $j = d^-(v)$ for $v \in V(S)$. Suppose e and f are positive and negative edges of S, respectively. Now, we have to show that j is even and $i = j - 1$. Let if possible, j is odd. Then,

$$d_T^+(e) = 2i - 2, \tag{43}$$

$$d_T^+(f) = 2j \tag{44}$$

and

$$d_T^+(v) = i + j. \tag{45}$$

If i is even then $i + j$ is odd whereas $2j$ is even, then from Eqs. (44) and (45), $T(S)$ is not signed-regular. Next, if i is odd, then $2i - 2 \equiv 0 (mod\ 4)$. But j is odd, therefore from Eqs. (43) and (44), $T(S)$ is not signed-regular. Next, if j is even. Then,

$$d_T^+(e) = 2i \tag{46}$$

and

$$d_T^+(f) = 2j - 2. \tag{47}$$

If $i \neq j - 1$, then from Eqs. (46) and (47), $T(S)$ is not signed-regular. Thus by contraposition, conditions follow.

Sufficiency: Suppose for a (i, j)-signed-regular sigraph S, j is even and $i = j - 1$, where $i = d^+(v)$ and $j = d^-(v)$ for $v \in V(S)$. Suppose e and f are positive and negative edges of S, respectively. Then,

$$d_T^+(e) = 2i, \tag{48}$$

$$d_T^-(e) = 2j, \tag{49}$$
$$d_T^+(f) = 2j - 2, \tag{50}$$
$$d_T^-(f) = 2i + 2, \tag{51}$$
$$d_T^+(v) = 2i \tag{52}$$

and

$$d_T^-(v) = 2j. \tag{53}$$

Since $i = j - 1$, therefore Eqs. (48), (50) and (52) are equal as well as Eqs. (49), (51) and (53) are also equal. Thus, $T(S)$ is signed-regular. Hence the theorem. □

Example 16. Two sigraphs whose total sigraph is signed-regular, are shown in Figure 4.

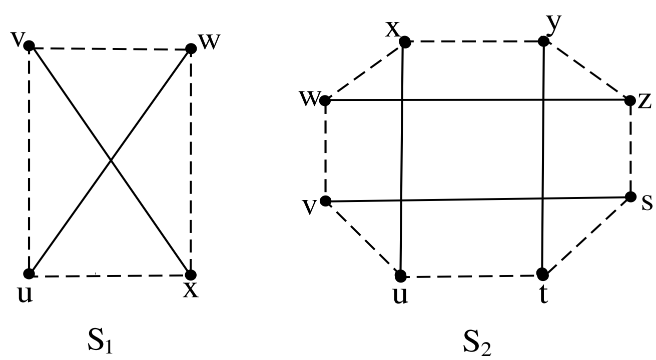

Figure 4: Sigraphs whose total sigraph is signed-regular.

Corollary 17. *The total sigraph of a heterogeneous (i, j)-signed-regular cycle is not a signed-regular sigraph.*

Proof. Since in the heterogeneous (i, j)-signed-regular cycle, $i = 1$ and $j = 1$. So by the Theorem 15, its total sigraph is not signed-regular. □

Corollary 18. *If the total sigraph $T(S)$ of a heterogeneous (i, j)-signed-regular sigraph $S = (S^u, \sigma)$, where S^u is of order n and degree $r = i + j$, is signed-regular, then $r \equiv 3(mod\ 4)$ and n is even.*

Proof. Due to Theorem 15, j is even and $i = j - 1$. Suppose $j = 2m, m \in Z^+$, then $i = 2m - 1$. It follows that $r = 4m - 1$. Thus, $r \equiv 3(mod\ 4)$. Since r is odd and S^u is regular, therefore n is even. □

5. Conclusion

In this paper, we have obtained the characterization of regularity of line sigraph, $\times$-line sigraph and total sigraph. Also, we have obtained some results related to energy of iterated line sigraphs.

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