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A Linear Algorithm for Resource Four-partitioning Four-connected Planar Graphs

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Abstract

Given a connected graph $G = (V, E)$, a set $V_r \subseteq V$ of r special vertices, four distinct base vertices $u_1, u_2, u_3, u_4 \in V$ and four natural numbers r_1, r_2, r_3, r_4 such that $\sum_{j=1}^{4} r_j = r$, we wish to find a partition V_1, V_2, V_3, V_4 of V such that V_i contains u_i and r_i vertices from V_r, and V_i induces a connected subgraph of G for each $i, 1 \leq i \leq 4$. We call a vertex in V_r a resource vertex and the problem above of partitioning vertices of G as the resource 4-partitioning problem. In this paper, we give a linear algorithm for finding a resource 4-partition of a 4-connected planar graph G with base vertices located on the same face of a planar embedding. Our algorithm is based on a 4-canonical decomposition and an st-numbering of G.

Keywords: Algorithm, 4-canonical decomposition, Four-partition, Four-connected graph, Planar graph, st-numbering, Resource bipartition, Resource four-partition.

2010 Mathematics Subject Classification: 05C85, 05C10.

1. Introduction

Let $G = (V, E)$ be a connected graph of $|V| = n$ vertices. Among these n vertices of G, some belong to a special class of vertices that we call *resource vertices*. Let $V_r \subseteq V$ be the set of resource vertices and $|V_r| = r$. Let $u_1, u_2, u_3, u_4 \in V$ be four designated vertices and r_1, r_2, r_3, r_4 be four natural numbers such that $\sum_{j=1}^{4} r_j = r$. Our goal is to find a partition V_1, V_2, V_3, V_4 of V such that $u_i \in V_i$, V_i contains r_i resource vertices and V_i induces a connected subgraph of G for each $i, 1 \leq i \leq 4$. We call this partitioning of vertices a *resource 4-partitioning* of G. For example, Figure 1(a) shows a connected graph G of $n = 19, r = 10$ vertices, where each resource vertex is drawn by a white circle. Figure 1(b) illustrates a resource 4-partition of G for $r_1 = 2, r_2 = 3, r_3 = 3, r_4 = 2$.

The resource 4-partitioning problem is a special case of resource k-partitioning problem, for $k = 4$. A *resource k-partitioning* is defined as partition $V_1, V_2, \ldots, V_k$ of V with a set $V_r \subseteq V$ of r resource vertices, base vertices $u_1, u_2, \ldots, u_k \in V, k$ natural numbers $r_1, r_2, \ldots, r_k$ such that $\sum_{j=1}^{k} r_j = r$, where $u_i \in V_i, V_i$ contains r_i resource vertices and V_i induces a connected subgraph of G for each $i, 1 \leq i \leq k$.

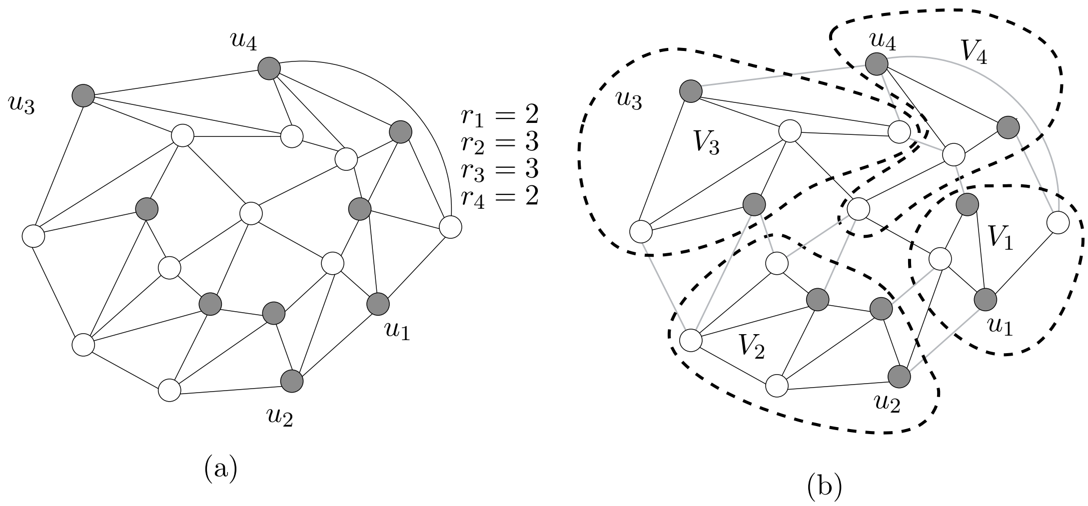

Figure 1: Resource 4-partitioning

Resource partitioning has significant applications in various areas. In computer network printers, routers, scanners etc. can be considered as resources. Resources require partitioning to balance loads on these resources and to prevent network traffic bottleneck. In multimedia network, it is required to assign a server to a specific group of clients for balancing loads among the servers. Resource partitioning has its application in the fault-tolerant routing of communication networks [15] and in computational aspects, too. For example, in grid computing we are required to divide a complex task such as computation of fractals into several subtasks and then we wish to delegate each of these subtasks to a computing element in the grid such that a computing element in the grid is not overwhelmed with tasks from other clients. This concept applies to telecommunication networks, fault tolerant systems, various producer-consumer problems and so on [1], [11].

A related problem is a *k-partitioning* problem in which we are given a graph $G = (V, E)$, k distinct base vertices $u_1, u_2, \ldots, u_k \in V$, and k natural numbers $n_1, n_2, \ldots, n_k$ such that $\sum_{j=1}^{k} n_j = |V|$, we wish to find a partition $V_1, V_2, \ldots, V_k$ of V such that $u_i \in V_i, |V_i| = n_i$ and V_i induces a connected subgraph of G for each $i, 1 \leq i \leq k$.

The k-partitioning problem is NP-hard in general [3]. Although not every graph has a k-partition, Györi and Lovász independently proved that every k-connected graph has a k-partition for any $u_1, u_2, \ldots, u_k$ and $n_1, n_2, \ldots, n_k$ [5], [8]. However, their proofs do not yield any polynomial time algorithm for actually finding a k-partition of a k-connected graph. For the cases $k = 2, 3, 4$ following algorithms have been known.

(i) There is a linear-time algorithm to find a bipartition of a biconnected graph [12, 13].

(ii) There is an $O(n^2)$ time algorithm to find a 3-partition of a triconnected graph [13].

(iii) There is a linear-time algorithm to find a 4-partition of a four connected planar graph with base vertices located on the same face of the given graph [10].

On the other hand, polynomial-time algorithms have not been known for the case $k \geq 4$. A polynomial-time algorithm for any k is claimed in [9], but is not correct [10]. If all the vertices are resource vertices then resource k-partitioning and k-partitioning problem are the same. Thus resource k-partitioning problem is also NP-hard [1]. The following algorithms are known for finding resource k-partitions of graphs for $k = 2, 3$.

(i) There are linear algorithms to find resource bipartitions of path-reducible graphs, series-parallel graphs and connected graphs where all resource vertices are contained in the same biconnected component [11].

(ii) There is an $O(n^2)$ algorithm to find vertex-subset tripartitions (equivalent to resource tripartitions [11]) of triconnected and 3-edge-connected graphs [14].

(iii) There is a linear algorithm to find a resource tripartition of a 3-connected planar graph [1].

But there exists no polynomial-time algorithm for resource k-partitioning of graphs for $k > 3$. In this paper we give a linear algorithm for finding a resource 4-partition of a 4-connected planar graph with base vertices located on the same face of a planar embedding. An early version of this paper is presented at [2]

The rest of the paper is organized as follows. Section 2 gives some definitions. In section 3, we present a linear algorithm for finding a resource 4-partition of a 4-connected planar graph with base vertices located on the same face of a planar embedding. Finally section 4 is a conclusion.

2. Preliminaries

In this section we define several graph theoretical terms used in this paper.

Let $G(V, E)$ be a connected simple graph with vertex set $V(G)$ and edge set $E(G)$. We denote by n the number of vertices in G and by m the number of edges in G. Thus $n = |V(G)|, m = |E(G)|$. An edge joining vertices u, v is denoted by (u, v). The *degree* of a vertex v in a graph G, denoted by $d(v)$, is the number of edges incident to v in G. The *connectivity* $\kappa(G)$ of a graph G is the minimum number of vertices whose removal results in a disconnected graph or a single-vertex graph K_1. We say that G is *k-connected* if $\kappa(G) \geq k$. A *walk*, $v_0, e_1, v_1, \ldots, v_{l-1}, e_l, v_l$, in a graph G is an alternating sequence of vertices and edges of G, beginning and ending with a vertex, in which each edge is incident to two vertices immediately preceding and following it. If the vertices $v_0, v_1, \ldots, v_l$ are distinct (except possibly v_0, v_l), then the walk is called a *path* and usually denoted either by the sequence of vertices $v_0, \ldots, v_l$ or by the sequence of edges $e_1, e_2 \ldots, e_l$. The *length* of a path is l which is one less than the number of vertices on the path. A path or walk is *open* if $v_0 \neq v_l$. A path or walk is *closed* if $v_0 = v_l$. A closed path containing at least one edge is called a *cycle*. For $W \subseteq V$, we denote by $G - W$ the graph obtained from G by deleting all vertices in W and all edges incident to them.

Let s and t be any two vertices of a connected graph G. An *st-numbering* of G is a numbering of its vertices by integers $1, 2, \ldots, n$ such that a vertex s receives number 1, a vertex t receives number n and every other vertex of G is adjacent to at least one lower-numbered vertex and at least one higher-numbered vertex. An interesting property of st-numbering of a graph is shown in the following fact.

(st1) If a graph G has an st-numbering $\pi = v_1, v_2, \ldots, v_n$, then both the subgraphs of G induced by $v_1, v_2, \ldots, v_i$ and $v_{i+1}, v_{i+2}, \ldots, v_n$ are connected for each $i, 1 \leq i \leq n$.

Not every connected graph has an st-numbering but the following Lemma [4] holds.

Lemma 2.1. *Let G be a biconnected undirected graph and (s, t) be any edge of G. Then G has an st-numbering $\pi = v_1, v_2, \ldots, v_n$ such that $v_1 = s$ and $v_n = t$, and π can be found in linear time.*

A graph is *planar* if it can be embedded in the plane so that no two edges intersect geometrically except at a vertex to which they are both incident. A *plane graph* is a planar graph with a fixed embedding. A plane graph divides the plane into connected regions called *faces*. We regard the contour of a face as a clockwise cycle formed by the edges on the boundary of the face. We denote the contour of the outer face of graph G by $C_o(G)$. We write $C_o(G) = w_1, w_2, \ldots, w_h, w_1$ if the vertices $w_1, w_2, \ldots, w_h$ on $C_o(G)$ appear clockwise in this order, as illustrated in Figure 2. We call a vertex an *outer vertex* and an edge an *outer edge*, if the vertex and edge respectively lie on $C_o(G)$.

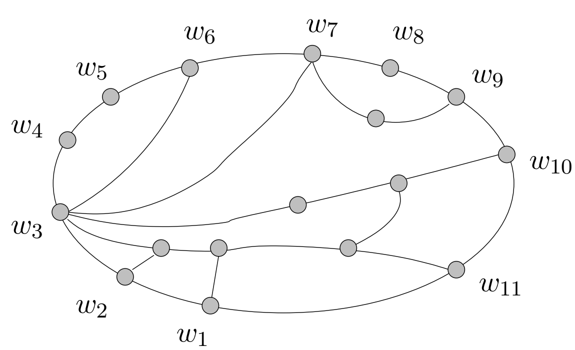

Figure 2: A plane graph with two outer chains

Let $\{v_1, v_2, \ldots, v_p\}, p \geq 3$, be a set of three or more outer vertices consecutive on $C_o(G)$ such that $d(v_1) \geq 3, d(v_2) = d(v_3) = \ldots = d(v_{p-1}) = 2$, and $d(v_p) \geq 3$. Then we call the set $\{v_2, v_3, \ldots, v_{p-1}\}$ an *outer chain* of G. The graph in Figure 2 has two outer chains $\{w_4, w_5\}$ and $\{w_8\}$.

The resource bipartitioning problem is a special case of resource k-partitioning problem, for $k = 2$. Let $V_r \subseteq V$ be the set of resource vertices and $|V_r| = r$. Let $u_1, u_2 \in V$ be two designated vertices and r_1, r_2 be two natural numbers such that $r_1 + r_2 = r$. Our goal is to find a partition V_1, V_2 of V such that $u_1 \in V_1, u_2 \in V_2$, V_i contains r_i resource vertices and V_i induces a connected subgraph of G for each $i, 1 \leq i \leq 2$. We call this partitioning of vertices a *resource bipartitioning* of G. We have the following Lemma on resource bipartitioning [11], [14].

Lemma 2.2. *A resource bipartition of a biconnected graph G can be found in linear time.*

We now give the definition of a *4-canonical decomposition* of a 4-connected plane graph [10]. Assume that $G = (V, E)$ is a 4-connected planar graph with four designated distinct vertices u_1, u_2, u_3, u_4 on the same face of G. We may assume that u_1, u_2, u_3, u_4 lie on the contour $C_o(G)$ of G, since, for any face F of G, we can re-embed G so that F becomes the outer face. We may furthermore assume that the four vertices u_1, u_2, u_3, u_4 appear on $C_o(G)$ of G in this order. Moreover we may assume that $(u_1, u_2), (u_3, u_4) \in E$; otherwise, consider as G the new graph obtained from G by adding edges (u_1, u_2) and (u_3, u_4). For a set $W_1, W_2, \ldots, W_i$ of pairwise disjoint subsets of V, we denote by G_i the subgraph of G induced by $W_1 \bigcup W_2 \bigcup \ldots \bigcup W_i$, and by $\bar{G}_i$ the subgraph of G induced by $V - W_1 \bigcup W_2 \bigcup \ldots \bigcup W_i$, that is, $\bar{G}_i = G - W_1 \bigcup W_2 \bigcup \ldots \bigcup W_i$. We say that a partition $\pi = W_1, W_2, \ldots, W_l$ of V is a 4-canonical decomposition of G if the following three conditions are satisfied.

- **(co1)** W_1 is the set of vertices on the inner face containing edge (u_1, u_2), and W_l is the set of vertices on the inner face containing edge (u_3, u_4),
- **(co2)** for each $i, 1 \leq i < 1$, both G_i and $\bar{G}_i$ are biconnected, and
- **(co3)** for each $i, 1 < i < 1$, either W_i consists of exactly one vertex on both $C_o(G_i)$ and $C_o(\bar{G}_{i-1})$ or W_i is an outer chain of G_i or $\bar{G}_{i-1}$.

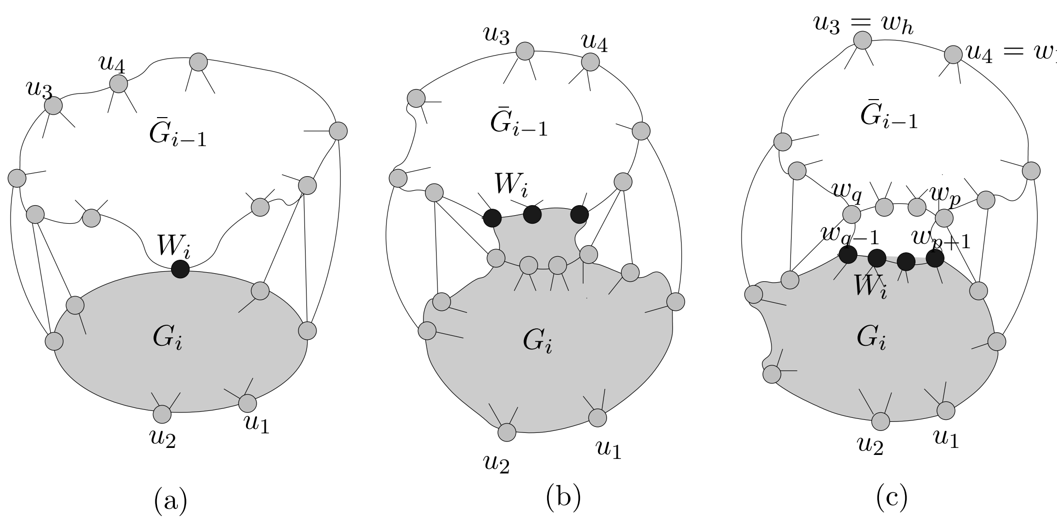

Figure 3: Illustration of the condition (co3).

Figure 3 illustrates the condition (co3); (a) for the case $|W_i| = 1$, (b) and (c) for the cases W_i is an outer chain of G_i and $\bar{G}_{i-1}$ respectively, where G_i and $\bar{G}_{i-1}$ are indicated by different shading and the vertices in W_i are drawn by black circles.

The 4-canonical decomposition [10] defined for 4-connected plane graphs is a generalization of the "canonical 4-ordering" defined for internally triangulated 4-connected plane graphs [6], [7]. We have the following two lemmas on 4-canonical decomposition [10].

Lemma 2.3. *Let $G = (V, E)$ be a 4-connected plane graph with four designated distinct vertices u_1, u_2, u_3, u_4 appearing on $C_o(G)$ in this order. Then G has a* 4*-canonical decomposition $\pi = W_1, W_2, \ldots, W_l$. Furthermore, π can be found in linear time.*

Lemma 2.4. *Let $W_1, W_2, \ldots, W_l$ be a* 4*-canonical decomposition of a* 4*-connected plane graph G. Then the following hold for any $i, 1 < i < 1$.*

(a) If W_i is an outer chain of G_i as illustrated in Figure 3(b), then, for any $W_i^{'} \subseteq W_i, \bar{G}_{i-1} - W_i^{'}$ is biconnected.

(b) If W_i is an outer chain of $\bar{G}_{i-1}$ as illustrated in Figure 3(c), then, for any $W_i^{'} \subseteq W_i, G_i - W_i^{'}$ is biconnected.

In Section 3, we provide a linear algorithm for finding a resource four-partition of a 4-connected planar graph with base vertices located on the same face of a planar embedding.

3. Resource Four-partition

In this section using a 4-canonical decomposition we give a linear algorithm for finding a resource four-partition of a planar graph with base vertices located on the same face of a planar embedding. We first bipartition the given 4-connected graph G into two almost biconnected subgraphs. Then each of them is bipartitioned into two connected subgraphs, and by adjusting the numbers of resource vertices in the resulting four subgraphs, a resource 4-partition of the given graph is obtained. To bipartition G into two closely biconnected subgraphs, a 4-canonical decomposition of the given graph is used.

Algorithm *resource_four-partition*
Input: A 4-connected planar graph $G = (V, E)$ with four designated distinct vertices u_1, u_2, u_3, u_4 appearing on the same face in this order and four natural numbers r_l, r_2, r_3, r_4 such that $\sum_{i=1}^{4} r_i = r$.
Output: A resource 4-partition of G.
begin
Take an embedding of G where u_1, u_2, u_3, u_4 appear on outer face $C_o(G)$ in this order.
Find a 4-canonical decomposition $\pi = W_1, W_2, \ldots, W_l$ of G;
Let i be the minimum integer such that $W_1 \cup W_2 \cup \ldots \cup W_i$ contains at least $r_1 + r_2$ resource vertices;

Let e be the excess number of resource vertices in $W_1 \cup W_2 \cup \ldots \cup W_i$ over $r_1 + r_2$;
There are the following two cases: (1) $e = 0$, and (2) $e \geq 1$;
Case 1: $e = 0$.
{ In this case, G_i contains $r_1 + r_2$ resource vertices, and $\bar{G}_i$ contains $r_3 + r_4$ resource vertices. }
Find a resource bipartition V_1, V_2 of the biconnected graph G_i such that $u_1 \in V_1, u_2 \in V_2$, V_1 contains r_1 resource vertices and V_2 contains r_2 resource vertices, and both V_1, V_2 induce connected subgraphs;
Find a resource bipartition V_3, V_4 of the biconnected graph $\bar{G}_i$ such that $u_3 \in V_3, u_4 \in V_4$, V_3 contains r_3 resource vertices and V_4 contains r_4 resource vertices, and both V_3, V_4 induce connected subgraphs;
return V_1, V_2, V_3, V_4 as a resource 4-partition of G.
Case 2: $e \geq 1$.
{ In this case, G_i contains $r_1 + r_2 + e$ resource vertices, and $\bar{G}_i = \bar{G}_{i-1} - W_i$ contains $r_3 + r_4 - e$ resource vertices. Since $e \geq 1$,W_i contains at least 2 resource vertices, $|W_i| \geq 2$ and hence W_i is an outer chain of either $\bar{G}_{i-1}$ or G_i. }
Let $C_o(\bar{G}_{i-1}) = w_1, w_2, \ldots, w_h, w_1$ where $w_1 = u_4$;
Assume that $W_i = \{w_{p+1}, w_{p+2}, \ldots, w_{q-1}\}$ is an outer chain of $\bar{G}_{i-1}$ as illustrated in Figure 3(c), otherwise, interchange the roles of u_1, u_2 and u_3, u_4;
Find an st-numbering $v_1, v_2, \ldots, v_z$ of $\bar{G}_i$ such that $s = v_1 = u_4$ and $t = v_z = u_3$;
Let $w_p = v_{p'}$ and $w_q = v_{q'}$;
Assume that $p' < q'$, otherwise, interchange the roles of u_3 and u_4;
Let $v_1, v_2, \ldots, v_{p'}$ contains x resource vertices;
There are the following three subcases: (a) $r_4 \leq x$, (b) $x + e \leq r_4$, and (c) $x < r_4 < x + e$;
Subcase 2a: $r_4 \leq x$.
{ In this subcase, the last e' vertices containing e resource vertices in the outer chain W_i are added to $\bar{G}_i$ as the deficient e vertices. }
Let $V_4 = \{v_1, v_2, \ldots, v_{n_4}\}$ be the first n_4 vertices containing r_4 resource vertices in the st-numbering of $\bar{G}_i$;
Let $V_3' = \{v_{n_4+1}, v_{n_4+2}, \ldots, v_z\}$ be the remaining vertices containing $r_3 - e$ resource vertices in $\bar{G}_i$;
{ By the fact (st1) of an st-numbering both V_4 and V_3' induce connected graphs. }
Let $W_i' = \{w_{q-1}, w_{q-2}, \ldots, w_{q-e'}\}$ be the set of the last e' vertices containing e resource vertices in W_i;
Let $V_3 = V_3' \cup W_i'$;
{ Since w_{q-1} is adjacent to $w_q \in V_3', V_3$ induces a connected graph with r_3 resource vertices. }
Let $G_{12} = G_i - W_i'$;
{ G_{12} is biconnected by Lemma 2.4(b), and has $r_1 + r_2$ resource vertices. }
Find a resource bipartition V_1, V_2 of G_{12} such that $u_1 \in V_1, u_2 \in V_2, V_1$ contains r_1 resource vertices, and V_2 contains r_2 resource vertices, and both V_1, V_2 induce connected subgraphs;
return V_1, V_2, V_3, V_4 as a resource 4-partition of G.

Subcase 2b: $x + e \leq r_4$.
{In this subcase, the first e' vertices containing e resource vertices in W_i are added to $\bar{G}_i$ as the deficient e resource vertices. }
Let $V_4' = \{v_1, v_2, \ldots, v_{n_4}\}$ be the first n_4 vertices containing $r_4 - e$ resource vertices in the st-numbering of $\bar{G}_i$, where $w_p = v_{p'} \in V_4'$;
Let $V_3 = \{v_{n_4+1}, v_{n_4+2}, \ldots, v_z\}$ be the remaining vertices containing r_3 resource vertices in $\bar{G}_i$;
Let $W_i' = \{w_{p+1}, w_{p+2}, \ldots, w_{p+e'}\}$ be the set of the first e' vertices containing e resource vertices in W_i ;
Let $V_4 = V_4' \cup W_i'$;
{ Since w_{p+1} is adjacent to $w_p \in V_4', V_4$ induces a connected graph with r_4 resource vertices. }
Let $G_{12} = G_i - W_i'$;
{ G_{12} is biconnected by Lemma 2.4(b), and has $r_1 + r_2$ resource vertices. }
Find a resource bipartition V_1, V_2 of G_{12} such that $u_1 \in V_1, u_2 \in V_2, V_1$ contains r_1 resource vertices, and V_2 contains r_2 resource vertices, and both V_1, V_2 induce connected subgraphs;
return V_1, V_2, V_3, V_4 as a resource 4-partition of G.
Subcase 2c: $x < r_4 < x + e$.
{ In this subcase, $e \geq 2$; the first b vertices containing $r_4 - x$ resource vertices and the last c vertices containing $e - (r_4 - x)$ resource vertices in W_i are added to $\bar{G}_i$ as the deficient e resource vertices. }
Let $W_{i4}' = \{w_{p+1}, w_{p+2}, \ldots, w_{p+b}\}$ be the set of the first b vertices containing $r_4 - x$ resource vertices in W_{i4}';
Let $W_{i3}' = \{w_{q-1}, w_{q-2}, \ldots, w_{q-c}\}$ be the set of the last c vertices containing $e - (r_4 - x)$ resource vertices in W_{i3}';
{ Since $|W_{i4}' + |W_{i3}'| = b + c < |W_i|$, $W_{i4}' \cap W_{i3}' = \phi, |W_{i4}' \cup W_{i3}'| = b + c$ and $W_{i4}' \cup W_{i3}'$ contains e resource vertices. }
Let $V_4 = \{v_1, v_2, \ldots, v_{p'}\} \cup W_{i4}'$;
Let $V_3 = \{v_{p'+1}, v_{p'+2}, \ldots, v_z\} \cup W_{i3}'$;
{ V_3 and V_4 contains r_3 and r_4 resource vertices respectively, $w_p = v_{p'} \in V_4, w_q = v_{q'} \in V_3$, and hence both V_3 and V_4 induce connected subgraphs. }
Let $G_{12} = G_i - W_{i3}' \cup W_{i4}'$; { G_{12} is biconnected by Lemma 2.4 (b), and has $r_1 + r_2$ resource vertices. }
Find a resource bipartition V_1, V_2 of G_{12} such that $u_1 \in V_1, u_2 \in V_2, V_1$ contains r_1 resource vertices, and V_2 contains r_2 resource vertices, and both V_1, V_2 induce connected subgraphs;
return V_1, V_2, V_3, V_4 as a resource 4-partition of G.
end;

Clearly the running time of the above algorithm is $O(n)$. Thus we have the following theorem.

Theorem 3.1. *A resource 4-partition of any 4-connected planar graph G can be found in linear time if the four base vertices are located on the same face of G.*

4. Conclusion

In this paper, we present a linear-time algorithm for finding a resource four-partition of a 4-connected planar graph with base vertices located on the same face of a planar embedding. The attention of many researchers have been drawn on resource partitioning problem due to its practical applications. However, the following problems related to resource partitioning are still open.

(a) Developing algorithms for finding resource k-partitions of graphs for $k \geq 4$.

(b) Developing algorithms to find resource k-partitions of graphs for $k \geq 2$ where resources are specified for the partitions.

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