AKCE Int. J. Graphs Comb., 9, No. 1 (2012), pp. 31-42

Independent restricted domination and the line digraph

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Communicated by: T.W. Haynes
Received 08 November 2010; revised 24 October 2011; accepted 21 November 2011

Abstract

Let H be a digraph possibly with loops and let D be a digraph whose arcs are colored with the vertices of H (an H-colored digraph). A walk (path) P in D will be called an H-restricted walk (path) if the colors displayed on the arcs of P form a walk in H. An H-restricted kernel N is a set of vertices of D such that for every two different vertices in N there is no H-restricted path in D joining them, and for every x in $V(D) - N$ there exists an H-restricted path in D from x to N.

For the line digraph of D we consider its inner arc-coloration, defined as follows: If h is an arc of D with color c then any arc of the form (x, h) in $L(D)$ also has color c.

We prove that the number of H-restricted kernels in an H-colored digraph is equal to the number of H-restricted kernels in the inner coloration of its line digraph.

Keywords: Domination, arc-coloration, kernel, line digraph.

2010 Mathematics Subject Classification: 05C20, 05C38, 05C69.

1. Introduction

In 1932 Whitney [26] introduced the concept of the line graph. Several results are known about such graph [3, 18, 20]. In 1960 Harary and Norman [15] extended the concept of the line graph to digraphs (see also [2, 17, 18]). Specifically concerning with the concept of kernel, there are results about how kernels in a digraph (and some generalizations) are related with the kernels in its line digraph: M. Harminc [16] proved that the number of kernels in a digraph is equal to the number of kernels in its line digraph. H. Galeana-Sánchez and L. Pastrana Ramírez [11] proved the same for arc-colored digraphs and kernels by monochromatic paths. Lu-Qin et.al [21] and H. Galeana-Sánchez and Xuelinang-Li [10] separately studied the relation between (k, l)-kernels in a digraph and in its line digraph. The main result in this paper asserts that the number of H-restricted kernels in a digraph is equal to the number of H-restricted kernels in the inner coloration of its line digraph.

*Research of Delgado-Escalante supported by CoNaCyT grant No.170157

1.1. Terminology and notation

We use the standard terminology on digraphs as given in [4]. However we provide most of the necessary definitions and notation for the convenience of the reader. We also refer the reader to [1] for more notation and terminology. For a digraph D, the vertex set is denoted by $V(D)$ and the arc set by $A(D)$. If $S \subseteq V(D)$ is a nonempty set then the *subdigraph* of D *induced by the vertex set* S, $D[S]$, is that digraph having vertex set S, whose arc set consist of those arcs of D joining vertices of S. Likewise, if $F \subseteq A(D)$ is a nonempty set, then $D[F]$, the *subdigraph* of D *induced by the arc set* F, is the digraph with F as the arc set, whose vertices are those joined by the arcs in F. The arc $(z_1, z_2) \in A(D)$ is called an S_1S_2*-arc* if $z_1 \in S_1 \subseteq V(D)$ and $z_2 \in S_2 \subseteq V(D)$. The vertex z_2 is called the *terminal vertex* of the arc (z_1, z_2).

Let $z \in V(D)$. The set $N^+(z) = \{x \in V(D)$ such that $(z, x) \in F(D)\}$ is called the *exterior neighborhood of* z, while the set $N^-(z) = \{x \in V(D)$ such that $(x, z) \in F(D)\}$ is called the *interior neighborhood of* z. The cardinality of $N^+(z)$ and $N^-(z)$ are denoted by $\delta^+(z)$ and $\delta^-(z)$ respectively. $A^+(z) = \{(z, x) \in A(D)$ such that $x \in V(D)\}$ is the *arc exterior neighborhood of* z.

A set $I \subseteq V(D)$ is an *independent set* in D whenever $A(D[I]) = \emptyset$. If W is a path or a cycle in D then $\ell(W)$ will denote its length. If $\{z_1, z_2\} \subseteq V(D)$ then a z_1z_2*- walk* will denote a walk from z_1 to z_2 in D and if we restrict z_1 and z_2 to $V(W)$ then the z_1z_2*- walk* contained in W will be denoted by (z_1, W, z_2). If $I \subseteq V(D)$ and $z \in V(D)$ then a zI*-walk* is a zx-walk for some $x \in I$. By C_n we will denote the cycle of length n. Throughout this paper all paths and cycles are directed.
Consider an *arc-colored digraph* D. For an arc (z_0, z_1) of D we will denote by $c(z_0, z_1)$ its color. $F \subseteq A(D)$ is a *monochromatic set* if all of its arcs are colored alike and $D' \subseteq D$ is a *monochromatic subdigraph* of an arc-colored digraph D if $A(D')$ is a monochromatic set.

1.2. Line digraph

The *line digraph* of $D = (X, U)$ is the digraph $L(D) = (U = V(L(D)), W = A(L(D)))$ whose vertices are the arcs of D, such that for every $h, k \in U$, $(h, k) \in W$ if and only if the terminal endpoint of h is the initial endpoint of k. In what follows, we denote the arc $h = (u, v) \in U$ and the vertex h in $L(D)$ by the same symbol.
If H is a subset of arcs in D then it is also a subset of vertices of $L(D)$. When we want to emphasize our interest in H as a set of vertices of $L(D)$, then we use the symbol H_L instead of H.
Now, let D be an arc-colored digraph and let $L(D)$ be its line digraph; the *inner arc-coloration of* $L(D)$ is the arc coloration of $L(D)$ defined as follows: If h is an arc of D with color c then any arc of the form (x, h) in $L(D)$ also has color c.

1.3. The background. Kernels and kernels by monochromatic paths

A *kernel* N of D is an independent set of vertices such that for each $z \in V(D) - N$ there exists a zN-arc in D. The concept of kernel was first introduced by Von Neumann and Morgenstern [25] in the context of Game Theory as an interesting solution for cooperative n-player games.

Notice that not every digraph has a kernel. Moreover when a digraph contains a kernel, it might not be unique. This simple observation compels us to ask for sufficient conditions for the existence of a kernel in a digraph. It is well known that if D is finite, the decision problem of the existence of a kernel in D is NP-complete for general digraphs [5, 24] as well as for planar digraphs with indegrees less than or equal to 2, outdegrees less than or equal to 2 and degrees less than or equal to 3 [7]. For any tighter constraints the problem is solvable in linear time. These results show how difficult the task of finding kernels can be.

In [8] Galeana-Sánchez introduced a generalization of the concept of kernel, the *kernel by monochromatic paths of an arc colored digraph* D, which is a set $N \subseteq V(D)$ satisfying the following conditions:

(a) N is an *independent set by monochromatic paths*: for every pair of different vertices u and v in N there is no monochromatic path between them in D; and

(b) N is an *absorbing set by monochromatic paths*: for every vertex $x \in V(D) - N$ there is a vertex $n \in N$ such that there is an xn-monochromatic path in D.

Several results arise around this concept, see [23], [13], [14] and [27], [28] and [29]. In particular, it is known about the relation between kernels by monochromatic paths of an arc colored digraph D and kernels by monochromatic paths in an specific arc coloration of its line digraph [11]: If D is an arc-colored digraph without monochromatic cycles, then the number of kernels by monochromatic paths in D is equal to the number of kernels by monochromatic paths in the inner arc coloration of $L(D)$.

1.4. The H-restricted coloration of a digraph and H-kernels

Let H be a digraph possibly with loops and let D be a digraph whose arcs are colored with the vertices of H, we call D an *H-colored digraph.* A walk (path) $P = (v_1, v_2, \ldots, v_n)$ in D will be called an H-restricted walk (path) if the colors displayed on P also form a walk (path) in H, i.e. $(c(v_1, v_2), c(v_2, v_3), \ldots, c(v_{n-1}, v_n))$ is also a walk in H.

We will say that a path of length one (an arc) is an H-restricted path.

Notice that when H consists only of loops then an H-restricted path in an H-colored digraph D is a monochromatic path. In the opposite case, if H has no loops and it has any other arc then an H-restricted path in D is an alternating path.

We will denote by

$$x \underset{H}{\rightarrow} y$$

an *H-restricted path from x to y in D*. Similarly we denote an *H-restricted path in D from a set of vertices S to a vertex y* by

$$S \underset{H}{\rightarrow} y$$

A walk W will be called of *Type A* if with at most one exception every path of length 2 in W is an H-restricted path.

The concept of an *H-restricted kernel* was introduced in [6] as the natural generalization of the two properties that define a kernel: A set $N \subseteq V(D)$ is called an H-restricted kernel if for every two different vertices in N there is no H-restricted path in D joining them (what is called an *H-independent set by restricted paths*), and for every x in $V(D) - N$ there exists an H-restricted path in D from x to N (an *H-absorbent set by restricted paths*). This new concept of an H-restricted kernel generalizes the concept of kernel by monochromatic paths.

It is important to mention that the concept of H-restricted paths and therefore that of H-restricted kernels has indeed its origins in the concept of monotonic paths introduced by Linek and Sands [19]. They assign a color to the arcs of a tournament with the vertices of a poset and say that a walk (path) is monotone if the colors displayed on it form a nondecreasing sequence in the partial order.

The concept of monotone paths came out of a result due to Sauer, Sands and Woodrow, who proved in [23] that if H is the reflexive, disconnected digraph with two vertices (the graph of the antichain of two elements) then the tournament coloring number of H, denoted by $tc(H)$, is one[1]. In other words, what the authors proved is that if the arcs of a tournament T are colored with two colors then there exists a single vertex x in T such that for every vertex $y \neq x$ in T there is a monochromatic path from y to x (i.e. there is an H-kernel of just one vertex). It is not even known if $tc(D)$ exists when H is the graph of an antichain of three or more elements. In [22] Reid discusses results concerning the tournament coloring number and explores it for reflexive digraphs with three vertices.

2. Main results

The two following lemmas show how H-restricted paths in an H-colored digraph can be preserved in its line digraph.

Lemma 2.1. *Let H be a digraph and let D be an H-colored digraph. Consider $\{x_0, x_n\} \subseteq V(D)$ and let $a_0 = (x, x_0)$ be an arc of D. If $T = (x_0, x_1, \ldots, x_{n-1}, x_n)$ is an H-restricted path in D then there exists an H-restricted path in the inner coloration of $L(D)$ from a_0 to a_n, where $a_n = (x_{n-1}, x_n) \in A(D)$.*

[1]For those readers not familiarized with this number, $tc(H)$ is the smallest positive integer (provided it exists) such that for any H-coloring of any tournament T, there exists a set S of at most $tc(H)$ vertices of T with the property of being an H-absorbent set.

Proof. For $i \in \{1, 2, \ldots, n\}$ let us denote by a_i the arc (x_{i-1}, x_i). Since T is a path in D, it follows from the definition of the line digraph that $(a_1, a_2, \ldots, a_n)$ is also a path in $L(D)$; actually the choice of a_0 implies that $(a_0, a_1, \ldots, a_n)$ is also a path in $L(D)$. Now, it follows from the definition of the inner coloration of the line digraph, that for every i, $1 \leq i \leq n$, it holds that (a_{i-1}, a_i) is colored in the same way as $a_i = (x_{i-1}, x_i) \in A(D)$ so, as $(c(x_{i-1}, x_i), c(x_i, x_{i+1})) \in A(H)$ (because T is an H-restricted path), we have that $(a_0, a_1, \ldots, a_n)$ is an H-restricted path in $L(D)$. □

Lemma 2.2. *Let H be a digraph and let D be an H-colored digraph which does not contain a closed Type A walk. Consider $\{a_0, a_n\} \subseteq V(L(D))$, let x_0 be the terminal endpoint of the arc a_0 in D and let x_n be the terminal endpoint of the arc a_n in D. If there exists an H-restricted path in $L(D)$ from a_0 to a_n, then $x_0 \neq x_n$ and there exists an H-restricted path in D from x_0 to x_n.*

Proof. Let $P = (a_0, a_1, \ldots, a_n)$ be an H-restricted path in the inner coloration of $L(D)$ and for $1 \leq i \leq n$ let us denote $a_i = (x_{i-1}, x_i)$ in D. For every i such that $0 \leq i \leq n-1$, we have that $(c(a_{i-1}, a_i), c(a_i, a_{i+1})) \in A(H)$ (because P is an H-restricted path in $L(D)$) and so $(c(a_i), c(a_{i+1}))$=$(c(x_{i-1}, x_i), c(x_i, x_{i+1})) \in A(H)$ (from the definition of the inner coloration). Then, since $(x_0, x_2, \ldots, x_n)$ is a walk in D (from the definition of the line digraph) we can conclude that it is an H-restricted walk in D. Now, since D has no closed walks of Type A it follows that $x_i \neq x_j$ for every $i \neq j$ such that $1 \leq i \leq n+1$ and $1 \leq j \leq n+1$; in particular $x_0 \neq x_n$ (notice that any closed H-restricted walk contains a closed walk of Type A) and so $(x_0, x_1, \ldots, x_n)$ is an H-restricted path in D. □

Now we will define a useful tool in the study of independent sets in line digraphs: For a digraph $D = (V, A)$, we denote by $\mathcal{P}(V)$ the set of all subsets of V and for each $Z \subseteq V$ we define the function

$$f : \mathcal{P}(V) \to \mathcal{P}(A) \text{ as}$$

$$f(Z) = \{(w, z) \in A \mid z \in Z\}.$$

This function was introduced by M. Harmic in [16] and it has been used in several results concerning independence in digraphs (and arc-colored digraphs as well) and the corresponding line digraph [10, 12, 11, 21, 9], because it has the nice property to map independent sets of vertices in D to independent sets of vertices in $L(D)$. The following lemma proves how the same property also holds for H-colored digraphs.

Lemma 2.3. *Let H be a digraph and let D be an H-colored digraph which does not contain a closed Type A walk. If $Z \subseteq V(D)$ is an H-independent set by restricted paths in D, then $f(Z)$ is also an H-independent set by restricted paths in the inner coloration of $L(D)$.*

Proof. We proceed by contradiction. Let $Z \subseteq V(D)$ be an H-independent set by restricted paths and suppose (by contradiction) that $f(Z)$ is not an H-independent set by restricted paths in the inner coloration of $L(D)$ (recall that $f(Z) \subseteq V(L(D)$ by the definition of the line digraph). Then there exist vertices h and k in $f(Z) \subset V(L(D))$ and an H-path between them in the inner coloration of $L(D)$. Let h' and k' be the terminal endpoints of the arcs h and k in D (they are well defined by the definition of the line digraph). By Lemma 2.2 we know that $h' \neq k'$ and there exists an H-restricted path between such vertices, a contradiction with the H-independence of Z (notice that $\{h', k'\} \subset Z$ from the definition of the line digraph). ☐

Now, we are able to show how a kernel in D is mapped onto another kernel in $L(D)$ and viceversa.

Theorem 2.4. *Let H be a digraph and let D be an H-colored digraph such that it does not contain a closed H-walk of Type A. The number of H-restricted kernels of D is equal to the number of H-restricted kernels in the inner coloration of $L(D)$.*

Proof. Let H be a digraph and $D = (V, A)$ be an H-colored digraph. Denote by $\mathcal{K}$ and by $\mathcal{K}_L$ the set of H-restricted kernels in D and in the inner coloration of $L(D)$, respectively.

Again we will use the function f previously defined for each $Z \subseteq V$:

$$f : \mathcal{P}(V) \to \mathcal{P}(A)$$

$$f(Z) = \{(w, z) \in A \mid z \in Z\} .$$

For a given $B \subseteq A$ consider the following sets:

$$C(B) = \{v \in V \mid \exists (z, v) \in B\}$$
$$I(B) = \{v \in V \mid \delta^-_D(v) = 0 \text{ and there is no } v \underset{H}{\to} C(B)\} .$$

Now, for a given $B \subseteq A$ we define the function

$$g : \mathcal{P}(A) \to \mathcal{P}(V)$$

$$g(B) = C(B) \cup I(B)$$

Note that $I(B)$ is required in the definition of g since there might be vertices in D such that they are not the terminal vertices of any arc in D.

Let f' be the restriction of f to $\mathcal{K}$ and let g' be the restriction of g to $\mathcal{K}_{\mathcal{L}}$. We will prove that every H-restricted kernel in D is mapped by f onto an H-restricted kernel in $L(D)$ and also every H-restricted kernel in $L(D)$ is mapped by g onto an H-restricted kernel in D. We will also prove that f' and g' are both injective functions:

Claim 1 *If $Z \in \mathcal{K}$, then $f(Z) \in \mathcal{K}_L$.*

Proof of Claim 1. Z is an H-independent set by restricted paths (since $Z \in \mathcal{K}$) so Lemma 2.3 implies that f(Z) is also an H-independent set by restricted paths in $L(D)$. Now we will prove that $f(Z)$ is an H-absorbent set by restricted paths. Let $t = (u, v) \in V(L(D)) - f(Z)$ so it follows from the definition of the function f that

$v \in V(D) - Z$. Since Z is an H-restricted kernel in D, then there exists an H-restricted path in D from v to some vertex $z \in Z$, let us say $(v = x_0, x_1, \ldots, x_n = z)$. Then Lemma 2.1 asserts that there is an H-restricted path in the inner coloration of $L(D)$ from (u, v) to $(x_{n-1}, x_n = z)$, where $(x_{n-1}, x_n = z) \in f(Z)$ (from the definition of f).

Claim 2 *f' is an injective function.*

Proof of Claim 2. Consider Z_1 and Z_2 two different H-restricted kernels in D. Without loss of generality suppose that $Z_1 - Z_2 \neq \emptyset$ and consider $z_1 \in Z_1 - Z_2$. Since Z_2 is an H-restricted kernel and z_1 is not in Z_2, there exist $z_2 \in Z_2$ and an H-restricted path from z_1 to z_2. Since Z_1 is an H-independent set by restricted paths, we find $z_2 \notin Z_1$. Let $h = (x_n, z_2)$ be the last arc of such restricted path. Then $h \in f(Z_2)$ (from the definition of the function f) and $h \notin f(Z_1)$ (because $z_2 \notin Z_1$) and this proves that $f(Z_1) \neq f(Z_2)$.

Claim 3 *If $B_L \in \mathcal{K}_L$, then $g(B_L) \in \mathcal{K}$.*

Proof of Claim 3. Independence The image under g of every H-restricted kernel in $L(D)$ is an H-independent set by restricted paths in D.

Let B_L be an H-restricted kernel in $L(D)$ and let u and v two different vertices in $g(B_L)$. In order to prove that there is no H-restricted path between u and v we must analyze the following cases:

Case i. $u, v \subseteq C(B_L)$.

Then u is the terminal vertex of and arc $a_u \in B_L$ and v is the terminal vertex of and arc $a_v \in B_L$. Suppose, by contradiction, that there exists an $u \underset{H}{\to} v$ in D, let us call it T. Let t be the last arc of T. If $a_v = t$, then it follows from Lemma 2.1 that there is an $a_u \underset{H}{\to} a_v$ in $L(D)$, a contradiction, since B_L is an H-independent set by restricted paths. In the other case if a_v is not t, then $t \notin B_L$, otherwise it follows from Lemma 2.1 that there exists an $a_u \underset{H}{\to} t$ in $L(D)$ which contradicts the H-independence of B_L. So there exist $b \in B_L$ and $P = (t = x_0, x_1, \ldots, x_n = b)$ an H-restricted path in the inner arc-coloration of $L(D)$. Notice that $b \neq a_v$ (Lema 2.2). Since t and a_v have the same end point, then it follows from the definition of the line digraph and its inner arc-coloration that $(a_v, x_1, \ldots, x_n = b)$ is an H-restricted path in the inner arc-coloration of $L(D)$, a contradiction (since a_v and b are vertices in B_L).

Case ii. $u, v \subseteq I(B_L)$.

Then $\delta_D^-(u) = 0 = \delta_D^-(v) = 0$ and so there is no H-restricted path toward v neither toward u, in particular there is no $u \underset{H}{\to} v$ in D and $u \underset{H}{\to} v$.

Case iii. *$u \in I(B)$ and $v \in C(B)$, or $v \in I(B)$ and $u \in C(B)$.* Without loss of generality, suppose that $u \in I(B)$ and $v \in C(B)$. Since there is no $u \underset{H}{\to} C(B)$ in D (from definition of $I(B)$), then there is no $u \underset{H}{\to} v$ in D. And since $\delta_D^-(u) = 0$ then there is no $v \underset{H}{\to} u$ in D.

Absorbtion The image of every H-restricted kernel in $L(D)$ under g is an H-absorbent set by restricted paths in D.

Let $B_L \in \mathcal{K}_{\mathcal{L}}$ and let $u \in V(D) - g(B_L) = V(D) - (c(B_L) \cup I(B_L))$. We want to exhibit an $u \underset{H}{\rightarrow} g(B_L)$ in D. Notice that u is not the terminal vertex of an arc in B_L (because $u \notin c(B_L)$). Since $u \notin I(B_L)$, then $\delta_D^-(u) > 0$ or there exists $u \underset{H}{\rightarrow} c(B_L)$ in D, in which case we do not have nothing to prove, since $c(B_L) \subseteq g(B_L)$. Suppose then that $\delta_D^-(u) > 0$ and consider an arc $(j, u) \in A - B_L$ (since there is no arc in B_L whose terminal vertex is u). B_L is an H-absorbent set by restricted paths in $L(D)$, hence there exists $(s, m) \in B_L$ and a $(j, u) \underset{H}{\rightarrow} (s, m)$ in $L(D)$. It follows from Lemma 2.2 that $u \neq m$ and that there exists an $u \underset{H}{\rightarrow} m$ in D. Finally notice that $m \in g(B_L)$ (since $(s, m) \in B_L$).

Claim 4 *g' is an injective function.*

Proof of Claim 4. Let M_L and N_L be two different H-restricted kernels in $L(D)$. Suppose that $M_L - N_L \neq \emptyset$ (the other case is analogous) and consider $h = (u, v) \in M_L - N_L$. Since v is the terminal vertex of an arc in M_L, $v \in c(M_L) \subseteq g(M_L)$. Now we will prove that $v \notin g(N_L)$. N_L is an H-absorbent set by restricted paths in $L(D)$ and $h = (u, v) \notin N_L$. So there exists $k = (x, y) \in N_L$ and an $h = (u, v) \underset{H}{\rightarrow} k = (x, y)$ in $L(D)$. It follows from Lemma 2.2, that $v \neq y$ and there exists a $v \underset{H}{\rightarrow} y$ in D. Since $y \in g(N_L)$ (because y is the terminal vertex of an arc in N_L) and $g(N_L)$ is an H-independent set by restricted paths, we have that $v \notin g(N_L)$. So $g(M_L) \neq g(N_L)$.

Finally notice that it follows from Claims 2 and 4 that $Card(\mathcal{K}) \leq Card(\mathcal{K}_{\mathcal{L}}) \leq Card(\mathcal{K})$ and hence $Card(\mathcal{K}) = Card(\mathcal{K}_{\mathcal{L}})$. □

3. Remarks

Remark 3.1. *Let H be a digraph, D be an H-colored digraph and let $L(D)$ be its line digraph. We can also define the outer arc-coloration of $L(D)$ as follows: If h is an arc of D with color c, then any arc (h, x) in $L(D)$ has color c. However Theorem 2.4 does not hold if we consider the outer arc-coloration of $L(D)$ instead of the inner arc-coloration.*

Proof. Consider the digraphs H and D in Figure 1 and $L(D)$ in Figure 2. D has not any closed H-restricted walk of Type A, D has an H-restricted kernel but the outer arc-coloration of $L(D)$ (see Figure 2) does not have an H-restricted kernel. □

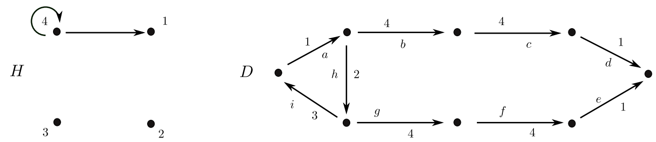

Figure 1:

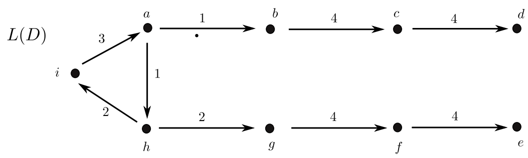

Figure 2:

Remark 3.2. *Theorem 2.4 does not hold if we drop the hypothesis that D has no any closed H-restricted walk of Type A.*

Proof. Consider the digraphs H and D in Figure 3 and $L(D)$ in Figure 4. D has a closed H-restricted walk of Type A and two H-restricted kernels, but the inner arc-coloration of $L(D)$ (see Figure 4) has just one H-restricted kernel. □

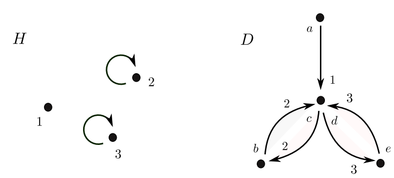

Figure 3:

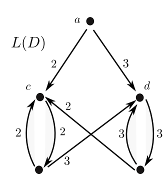

Figure 4:

Acknowledgements

The authors wish to thanks the anonymous referee for many comments which improved substantially the rewritting of this paper.

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